VARIANCE OF ELECTRICITY PRICES AND MARKET POWER WITH BILATERAL CONTRACTS IN DEREGULATED MARKETS

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DECLARATION

I hereby declare that the thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Wang Guanli 28 August 2012

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28 August 2012

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Contents

Summary

Many countries are in the process of reforming their electricity industries or are considering such reforms since the 1980s. The introduction of the electricity market is the most important part of this reform. This electricity market is considered as a deregulated electricity market compared to the regulated electricity prices. The deregulated electricity market is expected to be stable and competitive. However, price volatility and market power may exist in the deregulated electricity markets. To address these stabilization and competition issues, vesting contracts and forward contracts, which are both bilateral contracts, are introduced.

This thesis consists of four parts. The first part of the thesis is a literature review of market mechanism, bilateral contracts, price volatility, market power and oligopoly models.

The second part of the thesis describes how the vesting contracts work on controlling price volatility in the deregulated electricity market. The vesting contract is a kind of bilateral contract. A bilateral contract is an agreement on dispatching an amount of electricity (contract quantity) at a fixed price (contract price) during a certain time interval. Note that vesting contracts are imposed and not negotiated. The two basic elements of vesting contracts are hedge quantity and hedge price, which are similar to contract quantity and contract price of bilateral contracts. In the deregulated electricity market, the equilibrium price where supply and demand matches is called market clearing price (MCP) and the matched quantity is called market clearing quantity (MCQ). The customer price (CP) is a combination of hedge price and MCP weighted by their trading quantities. To study the impact of vesting contracts, we build mathematical models and analyze how the hedge price and hedge quantity affect the uncertainties of MCP and CP. Variances are used to characterize the uncertainties of MCP and CP. We assume that a generation company (genco) bids according to its Marginal Cost (MC) without considering vesting

contracts and supply function is uncertain in the mathematical models. We find that the variance of MCP increases when hedge quantity is assigned. However, the variance of CP decreases when hedge quantity is assigned. Also, a numerical study is conducted using the data of the Singapore electricity market from 2003 to 2010 to verify our models.

In the third part, supply function equilibria (SFE) and Cournot models are used to investigate the impact of bilateral contracts on the variances of MCP and CP. We assume that gencos bid strategically to maximize their profits while considering bilateral contracts and demand function is uncertain in this part. We find out that the variances of MCP and CP are decreasing functions of contract quantity in a competitive market by using the SFE model. Even when the market is not competitive, bilateral contracts can also reduce the variances of MCP and CP by setting contract quantity within a reasonable range in the SFE model. These two results, which hold in the SFE model, also hold in the Cournot model. Moreover, a numerical study is conducted to verify our models.

In the fourth part, we investigate the impact of bilateral contracts on the spot market by using the Cournot model. The MCQ, spot market quantity (SMQ), MCP, CP, profit of the market and market power in the spot market are examined closely. The SMQ is any amount of trading electricity other than contract quantity. We find three features in this part.

Firstly, we assume that demand function is changed with the introduction of bilateral contracts in our models. The analytical results show that our models are identical to those models with unchanged demand functions. This finding provides good justification of the assumption that demand function is unchanged with the introduction of bilateral contracts.

Secondly, we find some properties for the MCQ, SMQ, MCP, CP and profit of the market. When the bilateral contracts are introduced, MCQ may be increased and MCP may be decreased. We show that the MCQ is an increasing function of contract quantity. Also, the MCP and the SMQ are decreasing functions of contract quantity. We also show that MCQ with contracts is an upper bound of MCQ without contracts, and MCQ without contracts is an upper bound of SMQ. Moreover, we show that the MCP is reduced in the spot market with contracts. The variances of MCP are identical with and without bilateral contracts. However, the variance of CP is reduced with contracts. In addition, we find that the allocation of total contract quantity may not affect the MCQ, SMQ and MCP; that is, the allocation of fixed total contract quantity has no relationship with the MCQ, SMQ and MCP. Besides, we find several properties for the profit of the market. We derive the closed forms for total profit of the market with and without contracts. We also show that the total profit of the market is reduced by the introduction of bilateral contracts if contract price is less than MC.

Thirdly, the impact of bilateral contracts on the market power is investigated. We first use a conventional index, Lerner Index, to test the market power. This Lerner Index shows that market power is reduced by the introduction of bilateral contracts. We then propose another index which is defined as the ratio of profits with and without competition. We call this index as the Profit Index. By using this Profit Index, we find that market power is an increasing function of contract price subject to a given contract quantity. A numerical study is conducted using the data of the Singapore electricity market from 2004 to 2010 to verify our analytical results.

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List of Abbreviations

CP	Customer Price
CR	Concentration Ratios
DM	Dominance Measure
EMA	Energy Market Authority
Genco	Generation Company
HHI	Herfindahl-Hirschman-Index
ISO	Independent System Operator
MC	Marginal Cost
MCP	Market Clearing Price
MCQ	Market Clearing Quantity
MRR	Must-run Ratio
MSCP	Market Surveillance and Compliance Panel
NETA	New Electricity Trading Arrangement
RMPI	Revenue-based Market Power Indicator
RSI	Residual Supply Index
SFE	Supply Function Equilibria
SMA	Supply Margin Assessment
SMQ	Spot Market Quantity
USEP	Uniform Singapore Energy Price

List of Notations

$$
\begin{aligned}
x &= \text{Total electricity quantity produced} \\
Q_s &= \text{Market clearing quantity (MCQ) without contracts, where } Q_s \geq 0 \\
P_s &= \text{Market clearing price (MCP) without contracts, where } P_s \geq 0 \\
P_s^c &= \text{Customer price (CP) without contracts, where } P_s^c \geq 0 \\
&= P_s \\
\bar{P} &= \text{Contract price, where } \bar{P} \geq 0 \\
\bar{Q} &= \text{Total contract quantity, where } \bar{Q} > 0 \\
\hat{Q}_s &= \text{MCQ with contracts, where } \hat{Q}_s \geq \bar{Q} \\
\omega &= \text{Hedge ratio, where } \omega \in (0, 1] \\
&= \bar{Q}/\hat{Q}_s \\
\hat{P}_s &= \text{MCP with contracts, where } \hat{P}_s \geq 0 \\
\hat{P}_s^c &= \text{CP with contracts, where } \hat{P}_s^c \geq 0 \\
&= \omega\bar{P} + (1 - \omega)\hat{P}_s
\end{aligned}
$$

Chapter 1

Introduction

Most electricity industries were vertically integrated and geographically monopolized before the 1980s (Joskow, 2008). However, these monopolies may be inefficient. Joskow (1998) pointed out the disadvantages, such as high operating costs, high prices and lack of new investment that existed in such a system. Thus, many countries are in the process of reforming their electricity industries or are considering the reforms, such as Argentina, Australia, Brazil, California (USA), Chile, New Zealand, Singapore, the Nordic countries and United Kingdom (Chang, 2007; Joskow, 2008). The main idea of the reforms is to introduce some market mechanism. With market mechanism, market clearing price (MCP) is determined by supply and demand. This kind of market is called the wholesale market. In this chapter, some background information about deregulated electricity markets is provided, followed by the motivation behind this research. We then present the scope of our study, and the contributions of our work. Finally, the thesis structure is provided.

1.1 Background

The electricity industries have been reformed since the 1980s in many countries, such as Chile, United Kingdom, California and Singapore (Joskow, 2008). Chile began its reform in 1982 (Arellano, 2008), which was the earliest electricity industry reform (Joskow, 2008). However, it did not consider market mechanism in the reform. Chile rearranged the capacity of each generating unit in an ascending production cost order. These reordered production costs can be considered as a supply function. Then, the electricity price was decided by this supply function and demand. The reform of Chile was incomplete as the electricity price was based on the production costs (Arellano, 2008). There were no offers submitted by generation companies (gencos).

Another example of reform is that of United Kingdom in 1988 (Joskow, 2008). The core of electricity reform in United Kingdom is the wholesale market. From April 1st, 1990 to March 26th, 2001, an electricity pool was set up and operated as the wholesale market. In each time period, gencos submitted their offers to the pool. Then, electricity was dispatched according to the offers and the actual demand in the pool. Payments to gencos were based on the price of marginal offer, which is the highest accepted offer (von der Fehr and Harbord, 1993). From March 27th, 2001, the electricity pool was replaced by the New Electricity Trading Arrangement (NETA). The NETA involves not only offers from gencos but also bids from customers. Moreover, each accepted offer is traded at its offer price instead of a uniform price (Green, 2003).

California started reforming its electricity industry in the middle of the 1990s (Green, 2003). The retailers in California were asked to supply electricity at fixed prices. However, retailers were also asked to purchase electricity from the wholesale market. If MCP is low, then this system works. Otherwise, retailers lose considerable amount of money (Kee, 2001). The California electricity crisis in 2000 and 2001 demonstrates that this type of system may be risky. Electricity MCP may be highly unstable in such a system.

Singapore began its electricity reform in 1995 (Energy Market Authority (EMA) Singapore, 2010b). The reform involves privatization of state-owned monopolies, reformation of regulations and reformation of the electricity market. A market was built and named as the New Electricity Market of Singapore in 2003. This market consists of two submarkets: a wholesale market and a retail market. The wholesale market also comprises of the procurement market and real-time market. The procurement market is for securing operation of the power system. In the real-time market, customers and gencos trade through Energy Market Company (Energy Market Authority (EMA) Singapore, 2008). This thesis studies the real-time part of the wholesale market. It is also called the electricity spot market. In the retail market, retailers buy electricity from the wholesale market and sell to consumers (Energy Market Authority (EMA) Singapore, 2010a).

The market mechanism of Singapore electricity spot market is similar to that of the pool in United Kingdom. For each half an hour, MCP is determined by offers of gencos, demands of customers and other system constraints. An offer includes two parts: quantity and price. They represent the electricity quantity a genco is willing to supply at that given price. By cumulating the quantities below a fixed price, the offers can be transferred into a supply function. The MCP is decided at the point where supply function intersects demand function.

1.2 Motivation

The core of the reformed electricity industries is the construction of electricity markets. These electricity markets are considered as deregulated electricity markets compared to the regulated electricity prices. The deregulated electricity markets are expected to be stable and competitive. However, the MCP may be unstable in the markets. For example, the California electricity crisis in 2000 and 2001 is caused by unstable electricity MCP. Usually, there are three reasons for unstable electricity MCP: unstable supply, unstable demand and high storage cost. The first reason is unstable supply. Unstable fuel oil prices and unforeseeable breakdown of generating units are the two causes of unstable supply. Sueyoshi and Tadiparthi (2008) attributed the California electricity crisis in 2000 and 2001 to the rising marginal production costs of crude oil and natural gas. In Singapore, 97% of generating units rely on fuel oil and natural gas to generate electricity (Market Surveillance and Compliance Panel (MSCP) Singapore, 2007). Since fuel cost takes a large proportion in electricity production, the prices of fuel oil and natural gas have significant influence on electricity MCP. Unforeseeable breakdown of generating units also contributes to unstable supply. Generating units may break down any time. Hence, gencos may not be able to supply the quantity they offer to sell in the wholesale market.

The second reason is unstable demand. The inelasticity and fluctuation are the two attributes of demand (Stoft, 2002). Demand is considered to be inelastic due to its lack of response to high price within a short time. Fluctuation of demand is caused by weather, temperature, unpredictable activities and other factors.

The third reason is that electricity storage cost is high, or that electricity storage may not be economically feasible. One common method for electricity storage is the pumped-storage hydroelectricity. However, the construction cost is extremely high and sometimes the method is infeasible due to the geographic environment, such as in Singapore. Thus, limited electricity storage may not be used to smooth the gap between supply and demand (Bessembinder and Lemmon, 2002). As a result, the MCP, which is mainly based on supply and demand, will become highly unstable if the gap between supply and demand cannot be smoothed (Anderson and Davison, 2008).

Apart from the stabilization, another issue of the deregulated electricity markets is the competitiveness of the markets. In a perfect competitive market, gencos bid according to their marginal costs. However, in a monopoly market or an oligopoly market, gencos adopt bidding strategies to maximize their own profits. This ability of a genco to use bidding strategies is called market power (Wolak, 2000). The genco with market power is usually called price maker (De La Torre, 2002). Generally, price makers can have influence on the electricity prices and earn more profits.

The deregulated electricity markets are expected to be competitive due to two reasons. The first reason is that competition encourages gencos to control operating costs and improve technologies in the spot markets. Secondly, the benefit of competition from the deregulated electricity markets can be shared by consumers (Joskow, 2008). However, market power does exist in some electricity markets. For example, Woo et al. (2003) examined the market power in electricity markets in the United Kingdom, Norway, Alberta and California. They found that market power existed in all these four markets. Mount (2001) showed that the gencos with market power can increase electricity prices. In this case, consumers suffer from high electricity prices.

In order to control the volatility of MCP and mitigate the market power, vesting and forward contracts are introduced. These two types of bilateral contracts work similarly in the deregulated electricity markets. The difference is that the forward contracts are negotiated and the vesting contracts are not. A bilateral contract is an agreement on dispatching an amount of electricity (contract quantity) at a fixed price (contract price) during a certain time interval. The contract price and quantity are the two basic elements of these contracts.

To sum up, stabilization and competition are the two important issues in the deregulated electricity markets. We are interested in exploring the impact of bilateral contracts on these two issues in this thesis. We hope the theoretical and empirical results of this thesis can benefit the deregulation process around the world to some extent.

1.3 Scope of study

In microeconomics, MCP is the equilibrium price decided by supply and demand. All the electricity traded is called market clearing quantity (MCQ). Other than the contract quantity, the balance trading quantity is called spot market quantity (SMQ). Bilateral contracts enforce the market to trade the contract quantity at contract price first. Other than the contract quantity, the remaining quantity is still traded at MCP, that is, SMQ is traded at MCP. As a result, a part of supply and demand has been satisfied by bilateral contracts. With contracts, participants face two trading prices: contract price and MCP. The combination of contract price and MCP (according to trading quantities) is called customer price (CP). We consider an unstable environment where supply and demand are unstable. Thus, MCP and CP are unstable. In the thesis, we study the uncertainties of MCP and CP with and without contracts. Variance is used to measure the uncertainty.

The main purpose of this thesis is to develop mathematical models to examine the stabilization and competition issues in the electricity markets. The specific objective of this research is to investigate the impact of bilateral contracts on the price volatility and market power.

To investigate the impact of bilateral contracts on the price volatility, we first develop analytical models to study the impact of vesting contracts on the variances of MCP and CP. In these analytical models, a genco supplies electricity according to its marginal cost without considering vesting contracts. In this case, the genco may lose or earn money from the vesting contracts. If the hedge price of vesting contract is higher than MCP, the genco earns money from the vesting contracts. Otherwise, it will lose money. Whether the genco is losing or earning money from the vesting contracts, the uncertainty of supplying the hedge quantity is shifted to the trading quantity other than the hedge quantity. As a result, the variance of MCP is increased. The crucial question is how the vesting contracts affect the variance of CP. We investigate this question closely in this thesis.

We then present two oligopoly models, supply function equilibria (SFE) and Cournot models, to investigate the impact of bilateral contracts on the variances of MCP and CP. In the SFE and Cournot models, gencos bid strategically by taking into consideration the production cost, demand and bilateral contracts. The goal of each genco is to maximize its own profit. The difference between oligopoly models and analytical models mentioned above is that the gencos bid strategically in the oligopoly models, while gencos bid according to marginal costs in the analytical models.

Furthermore, Cournot models are also used to examine the impact of bilateral contracts on the elements other than variances of MCP and CP in the deregulated electricity markets. These elements include the MCQ, SMQ and profit of the market.

To investigate the impact of bilateral contracts on market power, we propose an index using the data of profits to measure market power. That is because most existing indexes use only the data of market shares or market prices to measure market power. Although profit is directly used to measure market power, we are interested in studying the relative increase in profit to measure market power.

In this thesis, we consider an unstable environment where supply and demand are unstable. Variance is used to measure the uncertainty. Other risk measurement tools, such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), are left for future research. Furthermore, the models built are single-period models. Due to time constraints, multi-period models will be considered in future studies.

1.4 Contributions

There are three main contributions in this thesis. Firstly, we develop analytical models and analyze how the vesting contracts affect the uncertainties of MCP and CP. We consider an unstable environment and assume that supply is a discrete function. We find that the variance of MCP increases when hedge quantity is assigned. This result is consistent with the results of Sapio and Wylomanska (2008). However, the variance of CP decreases when hedge quantity is assigned. We also find that the variances of MCP and CP do not have relationships with hedge price. Moreover, we find that the variance of MCP is an increasing function of hedge quantity. A numerical study is conducted using data from the Singapore electricity market from 2003 to 2010 to verify our model assumptions and the main results. The data are also used to conduct parameter estimation.

Secondly, we present two models, SFE and Cournot, to investigate the impact of bilateral contracts on the variances of MCP and CP. We find out that the variances of MCP and CP are decreasing functions of contract quantity in a competitive market for the SFE model. Even when the market is not competitive, bilateral contracts can also reduce the variances of MCP and CP by setting contract quantity in a reasonable range in the SFE model. These two results, which hold in the SFE model, also hold in the Cournot model. Also, we show a numerical study based on Singapore electricity market to support our models.

Thirdly, we use Cournot models to investigate the impact of bilateral contracts on the spot market. Many researchers assume that demand function is not affected by the introduction of bilateral contracts in Cournot models (Niu et al., 2005; Bushnell 2007). However, we assume that demand function is affected by the introduction of bilateral contracts in our Cournot models. We found that the results of our models and the models of Niu et al. (2005) and Bushnell (2007) are identical. This finding provides good justification for the assumption that demand function is not affected by the introduction of bilateral contracts.

The MCQ, SMQ, MCP, CP, profit of the market and market power in the spot market are examined closely by Cournot models. When the bilateral contracts are introduced, MCQ may be increased and MCP may be decreased. We show that the MCQ is an increasing function of contract quantity. Also, the MCP and the SMQ are decreasing functions of contract quantity. We also show that MCQ with contracts is an upper bound of MCQ without contracts, and MCQ without contracts is an upper bound of SMQ. Moreover, we show that the MCP is reduced in the spot market with contracts. The variances of MCP are identical with and without bilateral contracts. However, the variance of CP is reduced with contracts. In addition, we find that the allocation of total contract quantity may not affect

MCQ, SMQ and MCP; that is, the allocation of fixed total contract quantity has no relationship with MCQ, SMQ and MCP. Besides, we find several properties for the profit of the market. We show the closed forms for total profit of the market with and without contracts. We also show that total profit of the market is reduced by the introduction of bilateral contracts if contract price is less than MC.

Lastly, the impact of bilateral contracts on the market power is investigated. We first use a conventional index, Lerner Index, to test market power. This Lerner Index shows that market power is reduced by the introduction of bilateral contracts. This result is consistent with the results of Kelman (2001) and Chang (2007). We then propose an index which is defined as the ratio of profit with and without competition. We call this index as the Profit Index. By using this Profit Index, market power is an increasing function of contract price for a given contract quantity. Several numerical studies are conducted using the data of the Singapore electricity market to verify our analytical results.

The results of this thesis have significant impact on using bilateral contracts to ensure a stable and competitive market environment. Moreover, the models built in this thesis are helpful for the market participants when they are signing bilateral contracts. Specifically, both the theoretical and empirical results can benefit the market participants in controlling their price uncertainties.

1.5 Organization of the thesis

This thesis focuses on two things. Firstly, we study the impact of bilateral contracts on the price volatility. Secondly, we study the market power in the deregulated electricity market. It consists of six chapters.

In Chapter 2, we present a literature review, which includes the market mechanism, bilateral contracts, price volatility, market power and oligopoly models. We first review these five areas separately. We also study the interaction of multiple areas.

In Chapter 3, we develop mathematical models and analyze how the vesting contracts affect the uncertainties of MCP and CP. A numerical study is conducted using data from the Singapore electricity market to verify our mathematical models. The data are also used to conduct parameter estimation.

In Chapter 4, we present two oligopoly models, SFE and Cournot, to investigate the impact of bilateral contracts on the variances of MCP and CP. We also implement a numerical study based on the Singapore electricity market to verify our models.

In Chapter 5, we study Cournot models and investigate the impact of bilateral contracts on the spot market. The MCQ, SMQ, MCP, CP and profit of the market in the spot market are examined closely. The impact of bilateral contracts on the market power is also investigated. We first use a conventional index, Lerner Index, to test the market power. Thereafter, we propose a new index called the Profit Index to measure market power.

Chapter 6 concludes the thesis. Directions for future research will also be discussed. One possible future study is to consider different risk measurement tools, such as VaR and CVaR. Another possible future study is multi-period models.

Chapter 2

Literature Review

The general structure of the electricity market is presented in some monographs (Wood and Wollenberg, 1994; Bhattacharya, et al., 2001; Stoft, 2002). In this chapter, we review the market mechanism, bilateral contracts, price volatility, market power and oligopoly models.

2.1 Review of market mechanism

The introduction of the electricity spot market is the core of the electricity industry reform, and electricity is traded in the electricity spot market. There are three participants in the electricity spot market: suppliers, customers and an independent system operator (ISO). Generally, the suppliers are gencos who sell electricity to customers. Customers may represent real consumers or retailers. Retailers buy electricity from the spot market and sell it to some real consumers. Usually, these real consumers do not have the economical scale to buy electricity directly from the spot market. Alternatively, it is not possible for them to buy electricity from the spot market in the initial stage of the deregulation. In many countries, the ISO collects information from both supply and demand sides, and then dispatches electricity for each period in the spot market, such as in Singapore and United Kingdom.

The trading periods may be different in different electricity spot markets. In Australia, Singapore and United Kingdom, each period is half an hour (Woo et al., 2003; Anderson et al., 2007; Energy Market Authority (EMA) Singapore, 2010a). However, each period is one hour in California, Norway and Spain (Woo et al., 2003; Baillo et al., 2004).

In this section, we first review the electricity supply and demand. Then, we present the trading procedures in the electricity spot market.

2.1.1 Supply

The electricity suppliers are gencos. The gencos bid to sell their electricity in the electricity spot market. They submit a set of offers. Each offer specifies the price at which the genco is willing to sell an amount of electricity.

Usually, the number of offers that each genco can submit is fixed in a specific electricity spot market. For example, the number that each genco can submit is 10 in Australia and Singapore (Hu et al., 2005; Energy Market Authority (EMA) Singapore, 2010a), 16 in California and only 3 in United Kingdom (Wang et al., 2008). Some researchers work on how the number of offers affects the bidding behaviors of gencos (Wang et al., 2008).

2.1.2 Demand

Usually, the demand side bidding is not available in the initial stage of the electricity spot markets. Demand is forecasted in each period as a single value in most electricity spot markets, such as in Australia and Singapore (Hu et al., 2005; Energy Market Authority (EMA) Singapore, 2010a). The reason is that short term demand is very inelastic in the electricity spot markets (Holmberg, 2008). Hence, the inelastic demand is often assumed in models of the electricity spot markets (von der Fehr and Harbord, 1993; Holmberg, 2008).

By assuming demand to be inelastic, the demand quantity is not affected by the market price. Due to the lack of demand response for inelastic demand, demand side bidding is strongly encouraged in the electricity spot markets. In some spot markets, demand side bidding is introduced just like supply side bidding. For example, demand side bidding is allowed in Norway, California and Spain electricity spot markets (Woo et al., 2003; Baillo et al., 2004). By considering demand side bidding, demand is usually formulated as a linear function in models of the electricity spot markets (Allaz and Vila, 1993; Bushnell, 2007).

2.1.3 Trading procedures

In an electricity spot market where demand side bidding is not allowed, the ISO collects the offers from supply side and forecasts the demand quantity. When demand side bidding is allowed, the ISO collects the offers from the supply side and the bids from the demand side. By ordering the offers and bids, both the supply and demand functions are available. Then, MCP is determined by the supply and demand. It is where the supply and demand matches. The ISO dispatches electricity according to the MCP. The offers with price lower than or equal to the MCP are accepted. Other offers are rejected in each period.

The pricing mechanism of the electricity spot market is used to decide the price at which electricity should be traded in each period. In general, there are two types of pricing mechanisms: uniform and pay-as-bid pricing mechanisms. With a uniform pricing mechanism, all the electricity is traded at a unique price, which is the highest price of all the accepted offers. Usually, this price is called MCP. Note that the uniform pricing mechanism is used in the Australia and Singapore electricity spot markets (Hu et al., 2005; Energy Market Authority (EMA) Singapore, 2010a). With a pay-as-bid pricing mechanism, each single offer is isolated and each trade is made according to its own offer price. The pay-as-bid pricing mechanism is adopted in the United Kingdom electricity spot market, which is called the NETA (Thomas, 2006).

Many researchers work on comparing the uniform and pay-as-bid pricing mechanisms. For example, Wolfram (1999b) argued that the switching from the uniform pricing mechanism to the pay-as-bid pricing mechanism may lead to inefficient production. Rassenti et al. (2003) found that the pay-as-bid pricing mechanism can reduce price volatility compared to the uniform pricing mechanism. However, the prices submitted in the pay-as-bid pricing mechanism are higher than the prices submitted in the uniform pricing mechanism. Xiong et al. (2004) compared these two pricing mechanisms under multi-agent scenarios. They provided experimental evidences to show that the pay-as-bid pricing mechanism may result in lower market prices and price volatility than the uniform pricing mechanism. Fabra et al. (2006) compared uniform and pay-as-bid pricing mechanisms in many scenarios, such as revenue, consumer surplus and productive efficiency. They showed that there is no obvious result on one pricing mechanism outperforming the other one.

Before submitting bids in the spot market, gencos and customers may sign contracts on dispatching an amount of electricity (contract quantity) at a fixed price (contract price). The signing of bilateral contracts is separated from the spot market and can be seen as financial instruments without any actual transfer of electricity.

Now, we review the electricity trading procedures as follows. Niu et al. (2005) and Bushnell (2007) showed that gencos and customers may sign bilateral contracts before the spot market. However, all the electricity should be traded at market price in the spot market first. After that, customers get credit from gencos if market price is greater than contract price; otherwise, customers pay debit to gencos. The credit/debit is calculated as:

$$\text{(market price - contract price)} \times \text{contract quantity},$$

where if the amount is positive, it is credit; otherwise, it is debit. As a result, contract quantity is sold at contract price. Any amount of electricity other than contract quantity is sold at market price. Besides, we are also concerned with the final price faced by customers. This price is called CP, which is combined by contract price and market price with trading-quantity weighted.

2.2 Review of bilateral contracts

Bilateral contracts are agreements between market participants to exchange electricity. It is an agreement involving an amount of electricity (contract quantity) and a fixed price (contract price) for a certain time interval. The agreement associates with a set of specified constraints, such as contract quantity, price, delivery time and duration (El Khatib and Galiana, 2007). Bilateral contracts usually have two forms: "futures" and "forward contracts". Generally, futures have several fixed formats and can be traded at any time on the secondary market until its delivery time (Hull, 1997). Forward contracts can be negotiated between gencos and customers directly. They usually do not join the secondary market (Hull, 1998).

Moreover, bilateral contracts can be either physical or financial (El Khatib and Galiana, 2007). By signing a physical bilateral contract, the genco does not need to actually produce the amount of electricity signed and sell it to the customer. However, by signing a financial bilateral contract, the genco does not need to actually produce the amount of electricity signed and the customer needs not to actually buy that electricity. A financial bilateral contract considers the difference between contract and market prices. If market price is higher than contract price, gencos pay back the difference to customers. Otherwise, customers pay the difference to gencos.

There are two types of bilateral contracts in the deregulated electricity markets: vesting and forward contracts.

2.2.1 Vesting contracts

The vesting contract is a type of financial bilateral contract, which contains hedge quantity, hedge price, delivery time and duration. Note that vesting contracts are imposed by government, and are not for negotiation. The introduction of vesting contracts is for the prevention of price volatility and market power in the initial stage of the deregulated electricity market (Energy Market Authority (EMA) Singapore, 2010a).

Many countries introduce vesting contracts into their deregulated electricity markets, such as Australia, New Zealand, Singapore and United Kingdom (Anderson et al., 2007; Chang, 2007). The vesting contracts are introduced into the deregulated Singapore electricity market in 2004 (Energy Market Authority (EMA) Singapore, 2007). A large portion of demand is hedged in the Singapore electricity market. In particular, 65% of the demand is covered from Quarter 1, 2004 to Quarter 2, 2007. From Quarter 3, 2007 to Quarter 1, 2011, 55% of the demand is covered. The level of vesting contracts in Singapore may be reduced in later years (Energy Market Authority (EMA) Singapore, 2010b).

There are two types of vesting contracts in the initial stage of the deregulated Australia electricity market (Chang, 2007). The first type of vesting contracts is the two way bilateral contract. If market price is greater than contract price, customers get credit from gencos. Otherwise, customers pay debit to gencos. The second type of vesting contracts is a one way bilateral contract with a floor price and a cap price. Usually, the cap price is much greater than the floor price. If the market price is less than the floor price, the gencos receive the market price plus the floor price. If the market price is between floor price and cap price, the gencos receive the market price. If the market price is more than the cap price, the gencos receive the cap price. Note that the vesting contracts are no longer used in Australia (Anderson et al., 2007).

Many researchers examine the effectiveness of vesting contracts on the deregulated electricity market. Wolak (2000) analyzed the impact of vesting contracts on the bidding strategies of gencos in the deregulated Australia electricity market. He found that the market prices decrease as the hedge quantities increase. Kee (2001) showed the advantage of vesting contracts in the initial stage of the deregulated electricity markets. He used California crises in 2000 and 2001, during which vesting contracts are not signed, as an example to discuss the advantage of vesting contracts. Ahn and Niemeyer (2007) also found that market prices decrease as the hedge quantities increase. They pointed out that vesting contracts can also reduce the incentive of new entry by setting an appropriate hedge price.

2.2.2 Forward contracts

The forward contract is a type of financial bilateral contracts which is signed voluntarily by gencos and customers. The difference between vesting and the forward contracts is that the forward contracts are negotiated and the vesting contracts are not. Since forward contracts are signed voluntarily, it forms an electricity forward contract market along with the electricity spot market.

Some research works consider the unit commitment problem in the deregulated electricity market. Valenzuela and Mazumdar (2003) and Kockar (2008) examined the unit commitment problem for gencos, when both spot market and forward contract market are available. Valenzuela and Mazumdar (2003) proposed a stochastic program for gencos to schedule their units with uncertain MCP to maximize their profits. Kockar (2008) considered the unit commitment problem with CO_2 emission cost.

Some participants with enormous electricity demand are called contestable customers (Energy Market Authority (EMA) Singapore, 2010a). Their behaviors may impact market prices. One example is Aluminum smelting industries. They consume a lot of electricity for Aluminum electrolysis, which is called Hall-Héroult process (Schwarz et al., 2001). Another example is the electricity retailers. The retailers provide electricity to many real consumers. The contestable customers have to bear the risks from demands and market prices. Instead, they a cumulate enormous demands and have the power to bargain for better forward contracts. These help them to manage the risk of market price. When both spot market and forward contract market are available, contestable customers can arbitrarily choose mixed sources to meet their demands. Their mixing strategies are widely investigated, such as Carrión et al. (2007a, 2007b).

The signing of bilateral contracts is one method used to control volatility of MCP. Kaye et al. (1990) showed that bilateral contracts allow participants to lock in a suitable price for electricity and avoid the adverse effects of price uncertainty. Chang and Park (2007) presented that the introduction of vesting contracts may reduce the variance of electricity prices in Singapore. Also, the signing of bilateral contracts can control the market power. Kelman et al. (2001) pointed out that the signing of bilateral contracts can reduce market power, which is measured by the data of market prices. Joskow and Kahn (2002) observed that gencos who do not exercise market power in the California electricity market had signed bilateral contracts for most of their output.

Generally, there are two types of gencos: price takers and price makers. The behaviors of price takers do not affect the electricity prices, while the behaviors of price makers do. Conejo et al. (2002) and Valenzuela and Mazumdar (2003) examined behaviors of price takers. De La Torre et al. (2002) examined how the bidding behaviors of price makers affect the market price. Besides, there are considerable literature examining the bidding behaviors of price makers when both spot market and forward contract market are available, for example, Mielczarski et al. (1999) and Niu et al. (2005).

2.3 Review of price volatility

In the deregulated electricity market, electricity prices depend on several factors, such as supply, demand, bilateral contracts and network conditions. The supply and the demand are volatile. Then, the resulting price based on the volatile supply and demand is also volatile. Price volatility refers to the unpredictable fluctuations of the electricity prices over time (Zareipour et al., 2007). The measurements of price volatility include price velocity, standard deviation of price returns, VaR and CVaR, as well as variance of MCP.

2.3.1 Price velocity

Price velocity is the daily average normalized hourly change in electricity price (Li and Flynn, 2004). Li and Flynn (2004) defined two types of price velocity. The first type of price velocity is the sum of absolute value of price change in one day as a fraction of the overall average price of the studied periods. The price change is the difference between period prices. The first type of price velocity is expressed as

$$\bar{V}_i = \frac{1}{m}\left\{\left[\left(\sum_{j=1}^{m-1}|p_{i,j+1}-p_{i,j}|\right)+|p_{i-1,m}-p_{i,1}|\right]/p^A\right\}, \quad \text{for} \quad i=1,2,\ldots,n,$$

where m is the number of periods in one day, n is the number of days being studied, $p_{i,j}$ is the price of period j in day i and p^A is the overall average price which is defined as

$$p^A = \frac{1}{m\times n}\sum_{i=1}^{n}\sum_{j=1}^{m}p_{i,j}.$$

The second type of price velocity is the sum of absolute value of price change in one day as a fraction of the average price of that day. It is expressed as

$$\hat{V}_i = \frac{1}{m}\left\{\left[\left(\sum_{j=1}^{m-1}|p_{i,j+1}-p_{i,j}|\right)+|p_{i-1,m}-p_{i,1}|\right]/p_i^A\right\}, \quad \text{for} \quad i=1,2,\ldots,n,$$

where p_i^A is the average price of day i which is defined as

$$p_i^A = \frac{1}{m}\sum_{j=1}^{m}p_{i,j}, \quad \text{for} \quad i=1,2,\ldots,n.$$

Li and Flynn (2004) used these two types of price velocities to examine 14 deregulated electricity markets. They found that the price velocities of these 14 markets vary widely.

2.3.2 Standard deviation of price returns

Volatility can also be measured by the standard deviation of price returns. There are two definitions of price return. The first definition of price return is the difference between actual price and expected price (Benini et al., 2004). The second definition of price return is the difference between prices over a period divided by the price at the beginning of the period (Zareipour et al., 2007). Standard deviation of price returns is expressed as

$$S = \sqrt{\frac{\sum_{i=1}^{n}(u_i - \bar{u})^2}{n-1}},$$

where n is the number of periods studied, u_i is the price return in period i and $\bar{u}$ is the mean of u_i for the n studied periods.

Benini et al. (2004) studied standard deviation of price returns in the Spain, California, United Kingdom and Pennsylvania-Jersey-Maryland (PJM) markets from 1999 to 2000. They found that the standard deviation of price returns increases as the electricity prices increase. Zareipour et al. (2007) examined both standard deviation of price returns and price velocity in the Ontario electricity market as well as the New England, New York and PJM electricity markets. The results showed that the price volatility in Ontario is higher than the price volatility in New England, New York and PJM.

2.3.3 Value-at-Risk and Conditional Value-at-Risk

Volatility can also be measured by VaR and CVaR. VaR is a measure developed by the financial industry. It measures the expected maximum loss over a certain time horizon within a given confidence interval. Given a confidence level $\alpha \in (0, 1)$, the VaR of an asset at the confidence level α is given by the smallest number x such that the probability that the loss X of this asset exceeds x is not larger than $1 - \alpha$. It is expressed as

$$V_\alpha = \inf\{x \in \mathbb{R} : P(X > x) \leq 1 - \alpha\},$$

where V_α is the VaR with confidence level α over a certain time horizon and $P(X > x)$ is the probability that the loss X exceeds x. Dahlgren et al. (2001) measured the price volatility by VaR to study the California electricity market during the summer of 2000. They proposed some remedies for the problems in the California electricity market.

CVaR is defined as the conditional expectation of losses given that the loss exceeds a threshold value (Alexander et al., 2006). It is also referred to as expected shortfall. CVaR is expressed as

$$V_\alpha^C = \frac{1}{\alpha}\int_0^\alpha V_\gamma d\gamma,$$

where V_α^C is the CVaR with confidence level α over a certain time horizon and V_γ is the VaR with confidence level γ over a certain time horizon.

2.3.4 Variance

Variance of MCP is a measure of how far away a set of prices are from each other. Many researchers study the variance of MCP in the electricity market. Mount (2001) studied price volatility by using the variance of MCP. He found that the price volatility decreases as the number of gencos increases. Chang and Park (2007) examined the variance of empirical electricity prices to investigate the market structures adopted in Singapore electricity market. Ruibal and Mazumdar (2008) considered the variances of equilibrium prices of the Cournot and SFE models. They found that the variance of equilibrium prices of the Cournot model decreases and the variance of equilibrium prices of the SFE model increases as the number of gencos increases.

Also, many researchers work on the impact of bilateral contracts on the price volatility. For example, Chang and Park (2007) studied the impact of vesting contracts on the volatility of MCP in Singapore electricity markets by using the standard deviation of price (square root of the variance). Sapio and Wylomanska

(2008) studied the impact of forward contracts on the price volatility measured by standard deviation of price returns.

2.4 Review of market power

In a monopoly market or an oligopoly market, gencos adopt bidding strategies to maximize their own profits. This ability of a genco to use bidding strategies is called market power (Wolak, 2000). The deregulated electricity markets are expected to be competitive. However, market power does exist in some electricity markets. For example, Woo et al. (2003) examined the market power in the United Kingdom, Norway, Alberta and California electricity markets. They found that market power exists in all these four markets. Mount (2001) showed that the gencos with market power can increase electricity prices. In this case, consumers suffer from high electricity prices. Other than the data of market prices, the data of market shares and profits are also used to measure market power. Generally, there are two types of indexes: structural and behavioral indexes.

2.4.1 Structural indexes

The structural indexes show the level of market concentration or the position of a genco in the market. These indexes include the k-firm concentration ratios (CR), Herfindahl-Hirschman-Index (HHI), supply margin assessment (SMA), residual supply index (RSI), dominance measure (DM) and must-run ratio (MRR). They can be found in much literature, such as Gan and Bourcier (2002), Chang (2007) and Melnik et al. (2008).

Concentration ratios (CR)

The CR is the sum of ratios of market share of the k largest gencos (Chang, 2007). It is expressed as

$$C_k^R = \sum_{i=1}^{k} s_i,$$

where s_i is the ratio of market share of genco i. Note that the market share is based on the generation capacity. From now on, when we refer market share, it is based on the generation capacity. The less the CR value is, the less the market concentration level is.

Herfindahl-Hirschman-Index (HHI)

The HHI is named after its inventors, Orris Clemens Herfindahl (Herfindahl, 1950) and Albert Otto Hirschman (Hirschman, 1945). It is the sum of squares of ratios of market share of all the gencos (Chang, 2007). It is expressed as

$$H = \sum_{i=1}^{n} {s_i}^2,$$

where s_i is the ratio of market share of genco i and n is the total number of gencos. The HHI value ranges from 1 to 100^2. Note that the less the HHI value is, the less the market concentration level is. According to the United States Horizontal Merger Guidelines (United States Department of Justice/Federal Trade Commission, 2010), a market with HHI value less than 1500 is considered to be free of market concentration. A market with HHI value between 1500 and 2500 is considered moderately concentrated and a market with HHI value greater than 2500 is considered highly concentrated.

Supply margin assessment (SMA)

The SMA is used to test whether market demand can be met without a certain genco (Chang, 2007). It is expressed as

$$U_i = D - Q_{-i},$$

where D is the market demand and Q_{-i} is the total generation capacity of the market other than the generation capacity of genco i. If U_i is positive, the genco i is pivotal in the market.

Residual supply index (RSI)

Similar to SMA, the RSI is also used to test whether a genco is pivotal in the market (Chang, 2007). It is expressed as

$$T_i = \frac{Q_{-i}}{D},$$

where D is the market demand and Q_{-i} is the total generation capacity of the market other than the generation capacity of genco i. If the ratio is less than 1, then, genco i is said to have market power. Note that the lower the ratio is, the higher the market power of genco i is. The SMA and RSI can be converted to each other once D is known.

Dominance measure (DM)

The DM is used to test whether a genco has a dominant position in the electricity markets (Melnik et al., 2008). The threshold is defined as

$$s^d = \frac{1}{2}\left[1 - \gamma(s_1 - s_2)\left(1 - \sum_{i=3}^{n} s_i\right)\right],$$

where $\gamma > 0$ is an exogenous parameter and s_i is the ratio of market share of the i's largest genco, for $i = 1, 2, \ldots, n$. The genco whose market share is greater than s^d is considered to have a dominant position.

Must-run ratio (MRR)

The CR, HHI, SMA, RSI and DM do not consider transmission constraints, which play a significant role in the ability of a genco to exercise market power. The MRR does consider the transmission constraints in different zones of the deregulated electricity markets (Gan and Bourcier, 2002). It is expressed as

$$M_i = \frac{D - I - Q_{-i}}{q_i},$$

where D is the demand and I is the import limit of the zone. Moreover, Q_{-i} is the total generation capacity of the market other than the generation capacity of genco i in the zone and q_i is the generation capacity of genco i in the zone. This index calculates the market power of a genco in a given zone. Note that the lower the MRR is, the lower the market power of the genco in the zone is.

Many researchers use structural indexes as measurements of the market power. For example, MRR is used to test the market power in the United Kingdom electricity market (Gan and Bourcier, 2002). Wang et al. (2004) discussed market power assessment in detail by using MRR. Chang (2007) applied CR, HHI, SMA and RSI to measure market power in the Singapore electricity market. Hellmer and Wårell (2009) used HHI and DM to measure market power in the Nordic electricity market.

2.4.2 Behavioral indexes

The behavioral indexes are directly generated from the data of market prices. They include Lerner Index and the variations of Lerner Index (Chang, 2007; Nanduri and Das, 2007). Also, the data of profits are directly used to measure market power. For example, Nanduri and Das (2007) proposed the revenue-based market power indicator (RMPI) which is generated from the data of profits.

Lerner Index

The Lerner Index considers the distance between market price and marginal cost (Chang, 2007). It is a relative difference defined as

$$\mathcal{L} = \frac{P - C}{P},$$

where P is the market price and C is the marginal cost. If $\mathcal{L}$ is 0, it means the market is a perfect competition market. Note that the lower the Lerner Index is, the lower the market power is.

Revenue-based market power indicator (RMPI)

Nanduri and Das (2007) proposed RMPI as a measurement of market power. It is the net profit (revenue minus cost) and is expressed as

$$\mathcal{E} = R - P_C,$$

where R is the revenue and P_C is the production cost. Note that the lower the RMPI is, the lower the market power is.

The Lerner Index and RMPI are widely discussed in literature. Wolfram (1999a) used Lerner Index to study the market power in the United Kingdom electricity market from 1992 to 1994. He showed that the empirical prices are not as high as the prices that the theoretical models predict. Puller (2007) examined the California electricity market from 1998 to 2000. He found that empirical prices are nearly as high as the prices predicted by the theoretical models. Chang (2007) used Lerner Index as well as some structural indexes to test market power in the Singapore electricity market. Nanduri and Das (2007) proposed a variation of Lerner Index, which uses average quantity weighted price instead of average price. They also used RMPI as a measurement of market power in their paper. Ciarreta and Esponosa (2009) proposed a lower bound of Lerner Index to test if market power exists in the Spain electricity market.

2.5 Review of oligopoly models

In this section, we review two different oligopoly models, Cournot and SFE models. Generally, gencos compete by making decisions on quantities in the Cournot models. In the SFE models, the gencos make decisions on supply function. Although the decision variables for Cournot and SFE models are different, these two models are both equilibrium models. That is, both models achieve Nash equilibrium when each genco's strategy is the best response to other genco's actual strategies (Ventosa et al., 2005).

2.5.1 The Cournot model

The Cournot model was first proposed by Cournot in 1838 (Cournot, 1838). In this model, gencos make the decision on the amount of electricity that they are willing to supply. The Cournot model is a well known oligopoly model studied by many researchers (Gibbons, 1992; Ventosa et al., 2005).

Without bilateral contracts, each genco maximizes its profit by making the decision on the quantity that it is willing to supply into the market. Then, the ISO dispatches electricity according to the offers from the gencos and the demand quantity. The problem of a single genco can be modeled as a two-level program (Hobbs et al., 2000, Vespucci et al., 2010). The first level is the genco decision problem which is modeled by the Cournot model. The second level is the ISO problem where the ISO obtains the market price by matching supply and demand. The first level problem can be solved by considering the Karush-Kuhn-Tucker optimality conditions, which are related to the second level problem. The two-level optimization problem is a Mathematical Program with Equilibrium Constraints (MPEC) problem. Since the problem of each genco can be formulated as an MPEC problem, the equilibrium among these MPEC problems is an Equilibrium Problem with Equilibrium Constraints (EPEC). Some algorithms are proposed to solve the EPEC (Hobbs et al., 2000; Vespucci et al., 2010).

Recently, some researchers have provided a model for both: the spot market and the forward market. The objective of gencos is to maximize their own profits in both markets. The forward contracts are signed before the spot market. Hence, this two market problem can be seen as a two-period problem (Yao et al., 2008). In the first period, the decision variable for each genco is the bilateral contracts. In the second period, the decision variable for each genco is the quantity that the genco is willing to supply into the spot market. From above, the problem of a single genco without considering bilateral contracts (the second period problem) can be formulated as an MPEC problem. As a result, these two markets are usually modeled as a two-period EPEC problem, in which each genco faces an individual MPEC problem.

Many researchers examine the effects of the existence of forward contracts on the Cournot models. With bilateral contracts, Allaz and Vila (1993) presented two suppliers Cournot models and investigated the impact of bilateral contracts on the market prices. In the model, they assume a linear demand function, the same quadratic production cost for suppliers and no arbitrage. The no arbitrage assumption says that the contract price is equal to the spot market price. Allaz and Vila (1993) showed that the forward market equilibrium can be obtained in closed form in their model. Bushnell (2007) extended this model and considered multiple gencos in the deregulated electricity market.

In Allaz and Vila (1993) and Bushnell (2007), production costs of gencos are assumed to be identical. For gencos with different production costs, Su (2007) showed the existence of optimal solutions for the deterministic two-period EPEC problem. Yao et al. (2008) modeled the two settlement electricity markets into an EPEC problem and introduced an algorithm which uses an iterative approach to solve the EPEC problem. Zhang et al. (2010) proposed a stochastic EPEC problem where market demand is uncertain. They discussed the relationship between spot and forward markets.

The Cournot model is also widely used to study the price volatility and the market power issues. Sapio and Wylomanska (2008) applied the Cournot model to study the price volatility which is measured by the standard deviation of price returns. They found that forward contracts significantly influence the price volatility in both theoretical analysis and simulation studies. For fixed bilateral contracts, the price volatility is increased by the introduction of these contracts. However, if bilateral contracts are allowed, the price volatility can be controlled by signing proper contracts. Also, the Cournot model is applied to analyze market power. Borenstein and Bushnell (1999) formulated the California market as a Cournot model. They found that the availability of hydroelectric production and the elasticity of demand are the two important factors on the market power.

There are limitations of the Cournot models. Generally, gencos' decisions are the quantities that they are willing to supply. However, in the electricity markets, gencos submit a set of quantity and price offers which cannot be captured in the Cournot model. To overcome the limitations of the Cournot model, there are two extensions: the conjectural variation and the conjectural supply function models (Day et al., 2002). In the conjectural variation model, a constant conjectural variation is introduced. It is defined as the first derivative of total quantities with respect to quantity of the genco under consideration. In the conjectural supply function model, the relationship between total quantities of all gencos and market price is formulated as a supply function.

2.5.2 The SFE model

The SFE model was first proposed by Klemperer and Meyer (1989). In the SFE model, each genco determines its own supply function. Its goal is to maximize its own profit. Bolle (1992) and Green and Newbery (1992) first applied the SFE model to the deregulated electricity market. Bolle (1992) investigated three different characteristics of the spot market by using the SFE model. Green and Newbery (1992) studied a SFE model where each genco supplies a smooth supply function. They used this model to find optimal supply functions in the United Kingdom electricity market. After that, the SFE models have been widely used to formulate the deregulated electricity markets.

In order to obtain the SFE, a set of differential equations needs to be solved. Thus, there may be no equilibrium or multiple equilibriums. There are some restrictions by which the number of equilibriums can be reduced. The first restriction is the capacity constraint (Green and Newbery, 1992; Newbery, 1998; Genc and Reynolds, 2010). This restriction is reasonable since all gencos have limited generation capacities. The second restriction is the cap price (Baldick and Hogan, 2002; Genc and Reynolds, 2010). The cap price policy specifies the maximum price that gencos can submit and is enforced in many deregulated electricity markets, such as Australia, California and Alberta (Woo et al., 2003; Hu et al., 2005; Anderson and Davison, 2008). The third restriction concerns the marginal cost. By assuming constant marginal cost (Newbery, 1998; Holmberg, 2007) or linear marginal cost (Klemperer and Meyer, 1989; Baldick et al., 2004), the number of equilibriums can be reduced. The fourth restriction is the uncertainty of demand (Klemperer and Meyer, 1989). Lastly, the supply functions that gencos can submit also affect the equilibriums (Baldick, 2002; Anderson and Xu, 2005).

Since it is difficult to calculate the equilibrium for general SFE, much literature considers symmetric and linear SFE models. For the symmetric SFE models, all gencos have the same production cost function. With demand uncertainty, Klemperer and Meyer (1989) assumed that symmetric gencos submit general supply functions and proved the existence of SFE. They also gave sufficient conditions for uniqueness of SFE. Rudkevich et al. (1998) studied the SFE model where symmetric gencos submit smooth supply functions. In their model, gencos have piecewise linear and convex cost functions. Moreover, demand is assumed to be inelastic and uncertain. Anderson and Philpott (2002) extended the model of Rudkevich et al. (1998) to symmetric gencos with arbitrary convex cost functions. However, the SFEs of Rudkevich et al. (1998) and Anderson and Philpott (2002) may be multiple. To have a unique SFE, more restrictions are needed. Holmberg (2008) showed the existence of a unique SFE under conditions of symmetric gencos, inelastic demand, cap prices and capacity constraints.

In the linear SFE models, each genco submits a linear supply function into the electricity spot market. Green (1996) considered linear SFE models where asymmetric gencos have linear marginal cost functions and demand is assumed to be linear. He applied this model to study the United Kingdom electricity spot market.

Baldick et al. (2004) extended the model of Green (1996) by allowing gencos to submit piecewise affine supply function and also applied it to the United Kingdom electricity market. They found that piecewise affine SFE solutions provided good matches to the empirical data of the United Kingdom electricity market. Rudkevich (2005) analyzed a SFE model similar to Green (1996) by using a learning process. He demonstrated that the learning process converges to a linear SFE. In a general SFE model, asymmetric gencos who have general production cost functions submit general supply functions. In this case, there are no perfect analytical results that can be used to calculate the general SFE. Hence, a numerical algorithm is developed to approximate the general SFE (Holmberg, 2009).

Many researchers examine the impact of bilateral contracts on the SFE. For example, Newbery (1998) studied the impact of bilateral contracts on the SFE of a linear SFE model. In the model, gencos have the same constant marginal cost and demand is linear. Niu et al. (2005) presented a linear SFE model where asymmetric gencos have linear marginal cost functions and demand is linear. They applied this SFE model to the electricity market of the Electric Reliability Council of Texas (ERCOT) and found that this model is able to capture features of bidding behaviors in the ERCOT market. Anderson and Xu (2005) studied the influence of the bilateral contract on the optimal supply functions. The constraints of their model are limited generation capacities and cap prices.

The SFE models are also used to study price volatility and market power issues. Ruibal and Mazumdar (2008) studied the variances of equilibrium prices of many models, including Cournot and SFE models. They found that the price variance of Cournot model decreases as the number of gencos increases. However, the price variance of SFE model increases as the number of gencos increases. The SFE model is also applied to analyze market power. Borenstein et al. (1995) estimated the market power by using Cournot and SFE models. They stated that the calculation of the equilibriums of Cournot and SFE models can provide an important indicator of the potential of market power. That is because the production costs of gencos and the demand function in the models are estimated by historical data. Green (1999) studied a supply function competition approach which originated with Klemperer and Meyer (1989) and adopted by Bolle (1992) and Green and Newbery (1992). He showed that gencos may hedge most of their output in the spot market with forward contracts.

In the spot electricity markets, the gencos are required to submit a finite set of offers (quantity and price pairs) which will result in a step function. However, numerous researchers study the continuous supply function. Thus, there is a gap between research studies of step supply function and continuous supply function. Holmberg et al. (2008) showed the step function converges to the continuous supply function as the number of offers increases.

2.6 Summary

In this chapter, we have reviewed five areas. They are market mechanism, bilateral contracts, price volatility, market power and oligopoly models. We have also examined the literature which studies the interaction among multiple areas. However, there is little or no research work regarding three interactions among multiple areas. Firstly, there is little literature regarding the mathematical analysis of the impact of vesting contracts on the price volatility. Thus, we attempt to build mathematical models and analyze how vesting contracts affect the uncertainties of MCP and CP in Chapter 3.

Secondly, many researchers focus on the equilibrium under different settings of the SFE and Cournot models. However, to the best of our knowledge, they did not examine the impact of bilateral contracts on the price variance in the SFE and Cournot models. In this thesis, we are interested in studying this problem.

Thirdly, indexes that are proposed to measure the market power do not capture the relative profit. The structural indexes use only the data of market shares, while the behavioral indexes use only the data of market prices and marginal costs. Although RMPI directly considers the data of profits, we are interested in finding the relative increase in profit to measure market power. Therefore, we propose an index using the data of profits to measure market power.

Chapter 3

Single Genco with Vesting Contracts

3.1 Introduction

Electricity industries were vertically integrated monopolies in most areas of the world before the 1980s (Joskow, 2008). However, these monopolies were inefficient. Joskow (1998) observed that high operating costs, high prices and lack of new investment existed in such systems. Thus, many countries reformed their electricity industries by the introduction of a market mechanism, such as Argentina, Australia, Brazil, California, Chile, New Zealand, Singapore, the Nordic countries and United Kingdom (Chang, 2007; Joskow, 2008).

Singapore began its electricity industry reform in 1995 (Energy Market Authority (EMA) Singapore, 2010a). Changes involve privatization of state-owned monopolies, adjustments to regulations and creation of an electricity market. A wholesale market was developed in 2003 (Energy Market Authority (EMA) Singapore, 2008). Every half an hour, an MCP is determined by offers, demands and other system inputs. Generally, this MCP is unstable due to high storage cost, unstable supply and unstable demand.

To provide a stable environment, the Singapore government uses vesting contracts as a risk management tool. These vesting contracts are bilateral contracts that contain values of hedge quantity and hedge price, delivery time and duration. The hedge price is the fixed price at which gencos are required to sell electricity and the hedge quantity is the fixed amount of electricity which gencos must sell at the hedge price. In Singapore, the vesting contracts are imposed by the government and are not negotiated.

In microeconomics, MCP is the equilibrium price determined by supply and demand. In a market with hedges, participants are required to trade the hedge quantity at the hedge price. This means that a portion of the supply and demand is satisfied by hedges. As a result, participants have two trading prices: the hedge price and the MCP. The trading-quantity weighted price is called CP. Chang and Park (2007) showed that the hedge price and hedge quantity may reduce the variance of MCP in Singapore by examining data.

However, an improper hedge price and quantity may discourage new investors and may be detrimental to participants' profit, thereby discouraging new investment (Ahn and Niemeyer, 2007). As a result, the wholesale market may become less competitive and efficient. Thus, we are interested in exploring and analyzing the impact of hedge price and quantity on the uncertainties of MCP and CP. Our goal is to study the effects of hedge prices and quantities and ensure a stable market environment.

There is little literature regarding the mathematical analysis of the effects of hedge price and hedge quantity. In this chapter, we consider an unstable environment where participants are required to trade the hedge quantity at the hedge price. In addition, participants offer the quantities and prices they are willing to buy or sell in the market. We assume that the supply is a discrete function of uncertainty and participants have zero expectation on the extra profit caused by the hedging price and quantity.

In Sections 3.2 and 3.3, we develop two mathematical models and analyze how the hedge price and quantity affect the uncertainties of MCP and CP. We discover that having hedge quantity increases the variance of MCP and simultaneously reduces the variance of CP. We also find that the hedge price has no effect on the variances of MCP and CP. In Section 3.4, we use data from the Singapore electricity market to test our models. Conclusions are presented in Section 3.5.

3.2 Analytical model 1

Supply and demand are the two critical factors that influence the wholesale electricity trading. The supply rises as the price rises. This positive relationship between price and supply is called the law of supply (McConnell and Brue, 2008). It is due to the increasing marginal production cost, i.e. the added cost of producing one more unit of electricity. In our model, we assume that price is a function of supply. This function is called the "supply function". Without loss of generality, we assume that the supply function is nondecreasing with quantity.

In Singapore, gencos provide offers to the market to denote the amount of electricity they offer at the corresponding price. Much research shows that the Singapore electricity market is workably competitive, such as Chang (2007) and Chang and Lee (2008). So, we believe that gencos make offers according to their production costs, generating unit availability and other related factors without strategic behavior. The production cost is uncertain due to uncertain fuel prices. Thus, we assume that the supply function is uncertain. The model assumes that gencos bid based on costs and does not consider unplanned generation outages. This is the limitation of our model.

Currently, there is no demand side bidding for electricity in Singapore. For each period, the demand is estimated based on historical data and unresponsive to market price (Energy Market Authority (EMA) Singapore, 2010a). This means that the realized historical demand for each period would be the same under any price scenario. Thus, we assume that the demand is inelastic without uncertainty in our model.

In the Singapore electricity market, offers are sorted based on their prices (Energy Market Authority (EMA) Singapore, 2010a). To supply a certain quantity, offers with lower prices are matched first. The equilibrium price where the supply and demand quantity matches in the market is called MCP and the matched quantity is called MCQ. In our model, the demand is a constant, which is derived from actual data. Thus, the MCQ is equal to demand quantity. The MCP is the price where the supply quantity is equal to the demand quantity. In the electricity market, all electricity is traded at MCP.

The Singapore government uses hedge price and hedge quantity as a risk management tool. Since a portion of electricity is required to be sold at the hedge price, gencos may lose money from a low hedge price or gain extra profit from a high hedge price. Define $\bar{P}$ and $\bar{Q}$ as the hedge price and quantity, respectively. Define R as the profit arising from the hedge price and quantity.

The gencos may consider some strategies to raise the market prices to sky high in order to maximize their expected profit. However, in the long run, the Singapore government expects to achieve competitively electricity prices in the electricity market (Energy Market Authority (EMA) Singapore, 2010a). The Singapore government may take actions to make participants' profit reasonable and to keep the market competitive and efficient. Hence, gencos have to consider the long-run equilibrium and the impact of these strategies is unknown. As a result, in a single period, gencos may consider a simple and fair strategy: having zero expectation on the extra profit caused by hedging price and quantity. In our model, we assume that gencos estimate the demand over and above the hedge quantity to be L and adjust their offer prices to balance the profit, R, with L for zero expectation. We call this procedure "neutralization". That is, we assume that the extra profit arising from the hedge price and quantity, R, will be neutralized by selling additional L quantity of electricity. Moreover, we assume that the changes of prices for all the additional L quantity of electricity are the same. Define h as the adjustment of prices to neutralize the profit R. We have $h = -R/L$. In Figure 3.1, the hedge price is set low. In this case, gencos receive negative profit to provide an amount of electricity equal to hedge quantity. They will neutralize this profit in the next L quantity. Hence, the prices of intervals are increased by h.

We also divide the supply quantity into three intervals: $I_1 = (0, \bar{Q}]$, $I_2 = (\bar{Q}, \bar{Q}+L]$, $I_3 = (\bar{Q}+L, \hat{Q}]$, where $\bar{Q}$ is the hedge quantity assigned by the government and $\hat{Q}$ is the generation capacity. Interval I_1 is the hedge interval, interval I_2 is the neutralization interval and interval I_3 is the interval after neutralization. Market forces imply that $0 < \bar{Q} < \hat{Q}$.

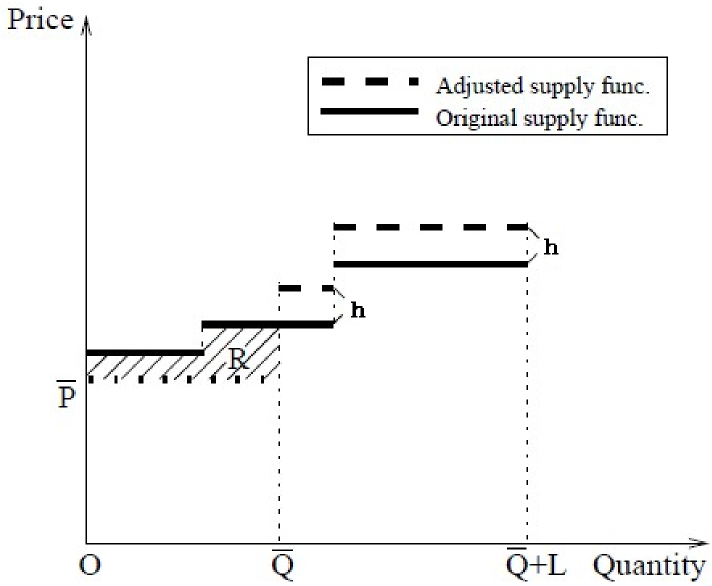

Figure 3.1: Neutralization of profit R

3.2.1 Notations

The following notations are used in this chapter.

Table 3.1: Notations used in analytical models 1 and 2

Q_i^s	$=$	Supply quantity of offer i, for $i = 1, 2, \ldots, m$,
		where $0 = Q_0^s < Q_1^s < \cdots < Q_m^s = \hat{Q}$
P_i^s	$=$	Supply price of offer i, for $i = 1, 2, \ldots, m$,
		where $P_1^s < P_2^s < \cdots < P_m^s$
a_i	$=$	$(Q_{i-1}^s, Q_i^s]$, for $i = 1, 2, \ldots, m$,
		where a_i is the quantity interval that offer i holds
A_i	$=$	$\|a_i\|$, for $i = 1, 2, \ldots, m$
x	$=$	Electricity quantity produced
$F(x)$	$=$	Original supply function
P_s	$=$	Market clearing price (MCP) without contracts, where $P_s \geq 0$
P_s^c	$=$	Customer price (CP) without contracts, where $P_s^c \geq 0$
	$=$	P_s
$\bar{P}$	$=$	Hedge price, where $\bar{P} \geq 0$
$\bar{Q}$	$=$	Hedge quantity, where $\bar{Q} > 0$
$\hat{Q}$	$=$	Generation capacity, where $\bar{Q} < \hat{Q}$
L	$=$	Estimated demand beyond hedge quantity,
		where $L > 0$ and $\bar{Q} + L$ is the estimated demand
I_1	$=$	$(0, \bar{Q}]$
I_2	$=$	$(\bar{Q}, \bar{Q} + L]$
I_3	$=$	$(\bar{Q} + L, \hat{Q}]$
R	$=$	Profit arising from hedge price and hedge quantity
$f(x)$	$=$	Adjusted supply function with hedge price and hedge quantity
Q_d	$=$	Actual demand quantity, where $Q_d \in (\bar{Q}, \hat{Q}]$
ω	$=$	Hedge ratio, where $\omega \in (0, 1]$
	$=$	$\bar{Q}/Q_d$
$\hat{P}_s$	$=$	MCP with contracts, where $\hat{P}_s \geq 0$
$\hat{P}_s^c$	$=$	CP with contracts, where $\hat{P}_s^c \geq 0$
	$=$	$\omega\bar{P} + (1 - \omega)\hat{P}_s$

3.2.2 Assumptions

We make the following three assumptions.

Assumption 3-1. Supply is a discrete function of uncertainty.

By reordering m offers with prices, the supply function is

$$F(x) = P_i^s + \varepsilon_i, \quad \text{for} \;\; x \in a_i, \tag{3.1}$$

where $a_i = (Q_{i-1}^s, Q_i^s]$ is the quantity interval that gencos are willing to supply subject to price $P_i^s + \varepsilon_i$. Note that $P_1^s < P_2^s < \cdots < P_m^s$ from the law of supply. Also, ε_i are identically distributed random variables with $Var(\varepsilon_i) = \sigma^2$, for $i = 1, 2, \ldots, m$.

Since most of the generating units in Singapore rely on fuel oil and natural gas, the uncertainties may be highly correlated. We assume that ε_i, for $i = 1, 2, \ldots, m$, are independent or pairwise positively correlated. That is, $\rho_{\varepsilon_i,\varepsilon_j} \in [0, 1]$, where $\rho_{\varepsilon_i,\varepsilon_j}$ is the correlation coefficient of ε_i and ε_j, for $i, j = 1, 2, \ldots, m$.

Assumption 3-2. Demand quantity is inelastic.

With no demand side bidding for electricity in Singapore, the demand is estimated based on historical data and may be unresponsive to market price in short period of time. In the model, we assume that the demand quantity is constant Q_d, independent of the price. Hence, the MCQ is equal to Q_d. Also, $Q_d \in (\bar{Q}, \hat{Q}]$. This implies that the demand is larger than the hedge quantity and no larger than the generation capacity. Further, $\bar{Q} \in a_{k_0}$ and $Q_d \in a_{k_1}$, where k_0 and $k_1 \in \mathbb{Z}$ and $1 \leq k_0 \leq k_1 \leq m$.

Assumption 3-3. With given hedge price and hedge quantity, the genco neutralizes R in interval I_2.

We assume that the adjusted supply function is parallel to the original supply function in I_2 and unchanged in I_3. With the given hedge price and hedge quantity, we define the adjusted supply function as

$$f(x) = \begin{cases} \bar{P}, & \text{for} \quad x \in I_1, \\ f_{I_2}(x), & \text{for} \quad x \in I_2, \\ f_{I_3}(x), & \text{for} \quad x \in I_3, \end{cases} \tag{3.2}$$

where

$$f_{I_2}(x) = P_i^s + \varepsilon_i + h, \qquad \text{for} \quad x \in a_i \cap I_2, \tag{3.3}$$

$$f_{I_3}(x) = F(x) = P_i^s + \varepsilon_i, \quad \text{for} \quad x \in a_i \cap I_3. \tag{3.4}$$

The constant $h = -R/L$ is the adjustment of offer prices in I_2, in order to neutralize revenue.

3.2.3 MCP without hedge price and hedge quantity

From Assumption 3-2, MCQ is equal to Q_d and $Q_d \in a_{k_1}$. Thus, the MCP is equal to $P_s = F(Q_d) = P_{k_1}^s + \varepsilon_{k_1}$ when no hedge price and hedge quantity are given. Consequently, the variance of MCP without hedge price and hedge quantity is

$$Var(P_s) = Var(P_{k_1}^s + \varepsilon_{k_1}) = Var(\varepsilon_{k_1}) = \sigma^2. \tag{3.5}$$

3.2.4 MCP with hedge price and hedge quantity

In this section, we show that the hedge quantity increases the volatility of MCP. We first examine the adjusted supply function when the hedge price, $\bar{P}$, and hedge quantity, $\bar{Q}$, are given. From Equation (3.2), the adjusted supply function, $f(x)$, is decided by hedge price, $\bar{P}$, and hedge quantity, $\bar{Q}$, in interval I_1. In interval I_2,

$$f(x) = f_{I_2}(x) = P_i^s + \varepsilon_i + h.$$

In order to find the adjusted supply function in I_2, we first explore h. From Assumption 3-3,

$$\begin{aligned} h &= -R/L \\ &= -\frac{\bar{P}\bar{Q} - \left[\sum_{i=1}^{k_0-1} A_i(P_i^s + \varepsilon_i) + (\bar{Q} - Q_{k_0-1}^s)(P_{k_0}^s + \varepsilon_{k_0})\right]}{L} \\ &= -\frac{\bar{P}\bar{Q} - \left[\sum_{i=1}^{k_0-1} A_i P_i^s + (\bar{Q} - Q_{k_0-1}^s)P_{k_0}^s\right]}{L} \\ &\quad + \frac{\sum_{i=1}^{k_0-1} A_i \varepsilon_i + (\bar{Q} - Q_{k_0-1}^s)\varepsilon_{k_0}}{L}. \end{aligned} \tag{3.6}$$

In interval I_3,

$$f(x) = f_{I_3}(x) = P_i^s + \varepsilon_i.$$

The MCP is equal to $\hat{P}_s = f(Q_d)$ when hedge price and hedge quantity are given. From Assumption 3-2, $Q_d \in (\bar{Q}, \hat{Q}]$. We consider two cases: $Q_d \in I_2$ and $Q_d \in I_3$.

Case 1. $Q_d \in I_2$.

From Equations (3.3) and (3.6), the MCP with hedge price and hedge quantity is

$$\begin{aligned} f_{I_2}(Q_d) &= P_{k_1}^s + \varepsilon_{k_1} - \frac{\bar{P}\bar{Q} - \left[\sum_{i=1}^{k_0-1} A_i P_i^s + (\bar{Q} - Q_{k_0-1}^s) P_{k_0}^s\right]}{L} \\ &\quad + \frac{\sum_{i=1}^{k_0-1} A_i \varepsilon_i + (\bar{Q} - Q_{k_0-1}^s)\varepsilon_{k_0}}{L}. \end{aligned} \tag{3.7}$$

Consequently, the variance of MCP with hedge price and hedge quantity is

$$\begin{aligned} Var(f_{I_2}(Q_d)) &= Var\left(\varepsilon_{k_1} + \frac{\sum_{i=1}^{k_0-1} A_i\varepsilon_i + (\bar{Q} - Q_{k_0-1}^s)\varepsilon_{k_0}}{L}\right) \\ &= \left(1 + \frac{\sum_{i=1}^{k_0-1} A_i^2 + (\bar{Q} - Q_{k_0-1}^s)^2}{L^2}\right)\sigma^2 + 2\sum_{\substack{i<j \\ i,j\in K}} C_i C_j \rho_{\varepsilon_i,\varepsilon_j}\sigma^2, \end{aligned} \tag{3.8}$$

where set $K = \{1, 2, \ldots, k_0\} \cup \{k_1\}$. Moreover, $C_i = A_i/L$, for $i = 1, 2, \ldots, k_0 - 1$, $C_{k_0} = (\bar{Q} - Q_{k_0-1}^s)/L$ and $C_{k_1} = 1$. Equation (3.8) holds because ε_i are identically distributed random variables, for $i = 1, 2, \ldots, m$.

Case 2. $Q_d \in I_3$.

Since $Q_d \in a_{k_1}$, the MCP with hedge price and hedge quantity is

$$f_{I_3}(Q_d) = P_{k_1}^s + \varepsilon_{k_1}. \tag{3.9}$$

Thus, the variance of MCP with hedge price and hedge quantity is

$$Var(f_{I_3}(Q_d)) = Var(P_{k_1}^s + \varepsilon_{k_1}) = Var(\varepsilon_{k_1}) = \sigma^2. \tag{3.10}$$

We now show that the variance of MCP does not decrease when the hedge quantity is introduced.

Theorem 3.1. *For $\rho_{\varepsilon_i \varepsilon_j} \in [0,1]$, $Var(\hat{P}_s) \geq Var(P_s)$.*

Proof. From Assumption 3-2, $Q_d \in (\bar{Q}, \hat{Q}]$. There are two cases: $Q_d \in I_2$ and $Q_d \in I_3$.

Case 1. $Q_d \in I_2$.

From Equations (3.5) and (3.8),

$$\begin{aligned} Var(f_{I_2}(Q_d)) &= \left(1+\frac{\sum_{i=1}^{k_0-1} A_i^2+(\bar{Q}-Q_{k_0-1}^s)^2}{L^2}\right)\sigma^2+2\sum_{\substack{i<j \\ i,j\in K}} C_i C_j \rho_{\varepsilon_i,\varepsilon_j}\sigma^2 \\ &> \sigma^2 \qquad (3.11) \\ &= Var(P_s), \end{aligned}$$

where Equation (3.11) holds because $\bar{Q} > Q_{k_0-1}^s$, $C_i \geq 0$ for all $i \in K$ and $\rho_{\varepsilon_i,\varepsilon_j} \geq 0$.

Case 2. $Q_d \in I_3$.

We have

$$Var(f_{I_3}(Q_d)) = \sigma^2 = Var(P_s), \qquad (3.12)$$

where Equation (3.12) holds from Equations (3.5) and (3.10).

From cases 1 and 2, since $\hat{P}_s = f(Q_d)$, we have $Var(\hat{P}_s) \geq Var(P_s)$. □

Next, we show other factors that affect the variance of the MCP.

Theorem 3.2. *For $Q_d \in I_2$ and $\rho_{\varepsilon_i \varepsilon_j} \in [0,1]$, $Var(\hat{P}_s)$ is a decreasing function of L, an increasing function of $\bar{Q}$ and independent of $\bar{P}$. Moreover, for $Q_d \in I_3$, $Var(\hat{P}_s)$ is independent of L, $\bar{Q}$ and $\bar{P}$.*

Proof. For $Q_d \in I_2$ and $\rho_{\varepsilon_i \varepsilon_j} \in [0,1]$, we have that $Var(f_{I_2}(Q_d))$ is a constant function of $\bar{P}$ from Equation (3.8). Similarly, $Var(f_{I_2}(Q_d))$ is a decreasing function of L because $\bar{Q}-Q_{k_0-1}^s \geq 0$, $A_i \geq 0$, for $i = 1, 2, \ldots, k_0-1$, and $\rho_{\varepsilon_i,\varepsilon_j} \geq 0$, for $i, j \in K$. Also, from Equation (3.8), $Var(f_{I_2}(Q_d))$ is an increasing function of $\bar{Q}$ because $A_i \geq 0$, for $i = 1, 2, \ldots, k_0-1$, and $\rho_{\varepsilon_i,\varepsilon_j} \geq 0$, for $i, j \in K$.

For $Q_d \in I_3$, we have that $Var(f_{I_3}(Q_d))$ is a constant function of L, $\bar{Q}$ and $\bar{P}$ from Equation (3.10). □

3.2.5 CP

In this section, we show that the volatility of MCP increased from the hedge quantity is not enough to overcome the stabilizing influence of the preset hedge price. As a result, the hedge quantity reduces the volatility of CP.

When the hedge quantities are zero, customers purchase all their electricity from the wholesale market at the MCP. Thus, the CP without hedge price and hedge quantity is $P_s^c = P_s$.

On the other hand, for positive hedge quantities, customers pay the hedge price, $\bar{P}$, for the hedge quantity, $\bar{Q}$. To meet the demand, customers buy the remaining $Q_d - \bar{Q}$ units of electricity from the wholesale market with MCP, $\hat{P}_s$. We define the CP as

$$\begin{aligned}\hat{P}_s^c &= \omega\bar{P} + (1-\omega)\hat{P}_s \\ &= \begin{cases}\omega\bar{P} + (1-\omega)f_{I_2}(Q_d), & \text{for } \ Q_d \in I_2,\\ \omega\bar{P} + (1-\omega)f_{I_3}(Q_d), & \text{for } \ Q_d \in I_3,\end{cases}\end{aligned} \tag{3.13}$$

where $\omega = \bar{Q}/Q_d$ is the hedge ratio and Equation (3.13) holds from Equation (3.2) for $Q_d \in (\bar{Q}, \hat{Q}]$.

We now consider two cases: $Q_d \in I_2$ and $Q_d \in I_3$.

Case 1. $Q_d \in I_2$.

The CP with hedge price and hedge quantity is

$$\begin{aligned}\hat{P}_s^c = \omega\bar{P} + (1-\omega)\Bigg(P_{k_1}^s + \varepsilon_{k_1} - \frac{\bar{P}\bar{Q}}{L} \\ + \frac{\left[\sum_{i=1}^{k_0-1} A_i P_i^s + (\bar{Q} - Q_{k_0-1}^s)P_{k_0}^s\right]}{L} \\ + \frac{\sum_{i=1}^{k_0-1} A_i\varepsilon_i + (\bar{Q} - Q_{k_0-1}^s)\varepsilon_{k_0}}{L}\Bigg),\end{aligned} \tag{3.14}$$

where Equation (3.14) holds from Equations (3.7) and (3.13). Similar to Equation (3.8), the variance of CP with hedge price and hedge quantity is

$$\begin{aligned} Var(\hat{P}_s^c) &= Var\left[\omega\bar{P} + (1-\omega)\left(P_{k_1}^s + \varepsilon_{k_1} - \frac{\bar{P}\bar{Q}}{L}\right.\right. \\ &\qquad + \frac{\left[\sum_{i=1}^{k_0-1} A_i P_i^s + (\bar{Q} - Q_{k_0-1}^s)P_{k_0}^s\right]}{L} \\ &\qquad \left.\left. + \frac{\sum_{i=1}^{k_0-1} A_i\varepsilon_i + (\bar{Q} - Q_{k_0-1}^s)\varepsilon_{k_0}}{L}\right)\right] \\ &= (1-\omega)^2\sigma^2\left(1 + \frac{\sum_{i=1}^{k_0-1} A_i^2 + (\bar{Q} - Q_{k_0-1}^s)^2}{L^2} + 2\sum_{\substack{i<j \\ i,j\in K}} C_i C_j \rho_{\varepsilon_i,\varepsilon_j}\right), \end{aligned} \tag{3.15}$$

where Equation (3.15) holds because ε_i are identically distributed random variables, for $i = 1, 2, \ldots, m$.

Case 2. $Q_d \in I_3$.

The CP with hedge price and hedge quantity is

$$\hat{P}_s^c = \omega\bar{P} + (1-\omega)(P_{k_1}^s + \varepsilon_{k_1}), \tag{3.16}$$

where Equation (3.16) holds from Equations (3.9) and (3.13). Thus, the variance of CP with hedge price and hedge quantity is

$$Var(\hat{P}_s^c) = (1-\omega)^2\sigma^2. \tag{3.17}$$

We now show that the variance of CP decreases when hedge quantity is assigned.

Lemma 3.1. *For $Q_d \in I_2$,*

$$(1-\omega)^2\left(1 + \frac{\sum_{i=1}^{k_0-1} A_i + (\bar{Q} - Q_{k_0-1}^s)}{L}\right)^2 \le 1.$$

Proof. For $Q_d \in I_2$, we have

$$\frac{\bar{Q}}{\bar{Q}+L} \le \frac{\bar{Q}}{Q_d} < 1,$$

where the inequality holds from $\bar{Q} < Q_d \le \bar{Q} + L$. Thus,

$$\begin{aligned} 0 &< 1 - \frac{\bar{Q}}{Q_d} \le 1 - \frac{\bar{Q}}{\bar{Q}+L}, \qquad \text{and} \\ 0 &< \left(1 - \frac{\bar{Q}}{Q_d}\right)^2 \le \left(\frac{L}{\bar{Q}+L}\right)^2. \end{aligned} \tag{3.18}$$

Hence,

$$\begin{aligned}(1-\omega)^2\left(1+\frac{\sum_{i=1}^{k_0-1}A_i+(\bar{Q}-Q^s_{k_0-1})}{L}\right)^2 &= \left(1-\frac{\bar{Q}}{Q_d}\right)^2\left(1+\frac{\bar{Q}}{L}\right)^2\\ &\leq \left(\frac{L}{\bar{Q}+L}\right)^2\left(\frac{L+\bar{Q}}{L}\right)^2\\ &= 1,\end{aligned}$$

where $\omega=\bar{Q}/Q_d$ and the inequality holds from Equation (3.18). $\square$

Theorem 3.3. *For* $\rho_{\varepsilon_i\varepsilon_j}\in[0,1]$, $Var(\hat{P}^c_s)\leq Var(P^c_s)$.

Proof. Consider two cases: $Q_d\in I_2$ and $Q_d\in I_3$.

Case 1. $Q_d\in I_2$.

From Equations (3.5) and (3.15),

$$\begin{aligned}Var(\hat{P}^c_s) &= (1-\omega)^2\sigma^2\left(1+\frac{\sum_{i=1}^{k_0-1}A_i^2+(\bar{Q}-Q^s_{k_0-1})^2}{L^2}+2\sum_{\substack{i<j\\ i,j\in K}}C_iC_j\rho_{\varepsilon_i,\varepsilon_j}\right)\\ &\leq (1-\omega)^2\sigma^2\left(1+\frac{\sum_{i=1}^{k_0-1}A_i+(\bar{Q}-Q^s_{k_0-1})}{L}\right)^2 \qquad (3.19)\\ &\leq \sigma^2 \qquad (3.20)\\ &= Var(P_s)\\ &= Var(P^c_s),\end{aligned}$$

where Inequality (3.19) holds because $\rho_{\varepsilon_i,\varepsilon_j}\leq 1$ for any $i,j=1,2,\ldots,m$. Inequality (3.20) holds from Lemma 3.1.

Case 2. $Q_d\in I_3$.

Since $0<\omega\leq 1$,

$$\begin{aligned}Var(\hat{P}^c_s) &= (1-\omega)^2\sigma^2\\ &< \sigma^2\\ &= Var(P_s)\\ &= Var(P^c_s),\end{aligned}$$

where the inequality holds from Equations (3.5) and (3.17). $\square$

Note that in Theorem 3.3, $Var(\hat{P}_s^c)$ is equal to $Var(P_s^c)$ if and only if $Q_d = \bar{Q} + L$ and $\rho_{\varepsilon_i,\varepsilon_j} = 1$ for any $i, j = 1, 2, \ldots, m$. Also, from Inequality (3.19), we learn that Theorem 3.3 holds for all correlation coefficients $\rho_{\varepsilon_i,\varepsilon_j} \in [-1, 1]$, where $i = 1, 2, \ldots, m$. That is, even if all random variables, ε_i, are not positively correlated, Theorem 3.3 is still true.

Next, we consider other factors that affect the variance of CP.

Theorem 3.4. *For $Q_d \in I_2$ and $\rho_{\varepsilon_i\varepsilon_j} \in [0, 1]$, $Var(\hat{P}_s^c)$ is a decreasing function of L and independent of $\bar{P}$. Moreover, for $Q_d \in I_3$, $Var(\hat{P}_s^c)$ is independent of L and $\bar{P}$.*

Proof. Similar to Theorem 3.2. $\square$

3.3 Analytical model 2

Since gencos make offers according to their production costs, more expensive generation units are being dispatched as demand is high. It is reasonable to assume that the uncertainty of offered price is associated with its price. In this section, we examine a case where the uncertainty of supply is a multiplied factor instead of an added factor, which is assumed in the Analytical model 1.

3.3.1 Assumptions

We modify the Assumptions 3-1 and 3-3 as the following.

Assumption 3-1′. Supply is a discrete function of uncertainty.

By reordering m offers with prices, the supply function is

$$F(x) = P_i^s(1 + \varepsilon_i), \quad \text{for } x \in a_i, \tag{3.21}$$

where $a_i = (Q_{i-1}^s, Q_i^s]$ is the quantity interval that gencos are willing to supply subject to price $P_i^s(1 + \varepsilon_i)$. Note that $P_1^s < P_2^s < \cdots < P_m^s$ from the law of supply. Also, ε_i are identically distributed random variables with $Var(\varepsilon_i) = \sigma^2$, for $i = 1, 2, \ldots, m$.

Since most of the generating units in Singapore rely on fuel oil and natural gas, the uncertainties may be highly correlated. We assume that ε_i, for $i = 1, 2, \ldots, m$, are independent or pairwise positively correlated. That is, $\rho_{\varepsilon_i,\varepsilon_j} \in [0, 1]$, where $\rho_{\varepsilon_i,\varepsilon_j}$ is the correlation coefficient of ε_i and ε_j, for $i, j = 1, 2, \ldots, m$.

Assumption 3-3′. With given hedge price and hedge quantity, the genco neutralizes R in interval I_2.

We assume that the adjusted supply function is parallel to the original supply function in I_2 and unchanged in I_3. With given hedge price and hedge quantity, we define the adjusted supply function as

$$f(x) = \begin{cases} \bar{P}, & \text{for } \ x \in I_1, \\ f_{I_2}(x), & \text{for } \ x \in I_2, \\ f_{I_3}(x), & \text{for } \ x \in I_3, \end{cases} \tag{3.22}$$

where

$$f_{I_2}(x) \;=\; P_i^s(1 + \varepsilon_i) + h, \qquad \text{for } \ x \in a_i \cap I_2, \tag{3.23}$$

$$f_{I_3}(x) \;=\; F(x) = P_i^s(1 + \varepsilon_i), \quad \text{for } \ x \in a_i \cap I_3. \tag{3.24}$$

The constant $h = -R/L$ is the shifting of offer prices in I_2 in order to neutralize revenue.

3.3.2 MCP without hedge price and hedge quantity

From Assumption 3-2, MCQ is equal to Q_d and $Q_d \in a_{k_1}$. Thus, the MCP is equal to $P_s = F(Q_d) = P_{k_1}^s(1 + \varepsilon_{k_1})$ when no hedge price and hedge quantity are given. Consequently, the variance of MCP without hedge price and hedge quantity is

$$Var(P_s) = Var(P_{k_1}^s(1 + \varepsilon_{k_1})) = {P_{k_1}^s}^2\sigma^2. \tag{3.25}$$

3.3.3 MCP with hedge price and hedge quantity

In this section, we show that the hedge quantity increases the volatility of MCP. We first examine the adjusted supply function when the hedge price, $\bar{P}$, and hedge quantity, $\bar{Q}$, are given. From Equation (3.22), the adjusted supply function, $f(x)$, is decided by hedge price, $\bar{P}$, and hedge quantity, $\bar{Q}$, in interval I_1. In interval I_2,

$$f(x) = f_{I_2}(x) = P_i^s(1+\varepsilon_i) + h.$$

In order to find the adjusted supply function in I_2, we first examine h. From Assumption 3-3$'$,

$$\begin{aligned} h &= -R/L \\ &= -\frac{\bar{P}\bar{Q} - \{\sum_{i=1}^{k_0-1} A_i[P_i^s(1+\varepsilon_i)] + (\bar{Q} - Q_{k_0-1}^s)[P_{k_0}^s(1+\varepsilon_{k_0})]\}}{L} \\ &= -\frac{\bar{P}\bar{Q} - \left[\sum_{i=1}^{k_0-1} A_i P_i^s + (\bar{Q} - Q_{k_0-1}^s)P_{k_0}^s\right]}{L} \\ &\quad + \frac{\sum_{i=1}^{k_0-1} A_i P_i^s \varepsilon_i + (\bar{Q} - Q_{k_0-1}^s)P_{k_0}^s \varepsilon_{k_0}}{L}. \end{aligned} \tag{3.26}$$

In interval I_3,

$$f(x) = f_{I_3}(x) = P_i^s(1+\varepsilon_i).$$

The MCP is equal to $\hat{P}_s = f(Q_d)$ when hedge price and hedge quantity are given. From Assumption 3-2, $Q_d \in (\bar{Q}, \hat{Q}]$. We consider two cases: $Q_d \in I_2$ and $Q_d \in I_3$.

Case 1. $Q_d \in I_2$.

From Equations (3.23) and (3.26), the MCP with hedge price and hedge quantity is

$$\begin{aligned} f_{I_2}(Q_d) &= P_{k_1}^s(1+\varepsilon_{k_1}) - \frac{\bar{P}\bar{Q} - \left[\sum_{i=1}^{k_0-1} A_i P_i^s + (\bar{Q} - Q_{k_0-1}^s)P_{k_0}^s\right]}{L} \\ &\quad + \frac{\sum_{i=1}^{k_0-1} A_i P_i^s \varepsilon_i + (\bar{Q} - Q_{k_0-1}^s)P_{k_0}^s \varepsilon_{k_0}}{L}. \end{aligned} \tag{3.27}$$

Consequently, the variance of MCP with hedge price and hedge quantity is

$$\begin{aligned}
Var(f_{I_2}(Q_d)) &= Var\left(P^s_{k_1}\varepsilon_{k_1} + \frac{\sum_{i=1}^{k_0-1} A_i P^s_i \varepsilon_i + (\bar{Q} - Q^s_{k_0-1})P^s_{k_0}\varepsilon_{k_0}}{L}\right) \\
&= \left({P^s_{k_1}}^2 + \frac{\sum_{i=1}^{k_0-1} A_i^2 P^{s2}_i + (\bar{Q} - Q^s_{k_0-1})^2 {P^s_{k_0}}^2}{L^2}\right)\sigma^2 \\
&\quad + 2\sum_{\substack{i=1,2,\ldots,k_0-1,\\ j=1,2,\ldots,k_0-1:\\ i<j}} \frac{A_i P^s_i A_j P^s_j \rho_{\varepsilon_i\varepsilon_j}\sigma^2}{L^2} \\
&\quad + 2\sum_{i=1,2,\ldots,k_0-1} \frac{A_i P^s_i(\bar{Q} - Q^s_{k_0-1})P^s_{k_0-1}\rho_{\varepsilon_i\varepsilon_{k_0}}\sigma^2}{L^2} \\
&\quad + 2\sum_{i=1,2,\ldots,k_0-1} \frac{A_i P^s_i P^s_{k_1}\rho_{\varepsilon_i\varepsilon_{k_1}}\sigma^2}{L} \\
&\quad + 2\frac{P^s_{k_1}(\bar{Q} - Q^s_{k_0-1})P^s_{k_0}\rho_{\varepsilon_{k_0}\varepsilon_{k_1}}\sigma^2}{L}, \qquad (3.28)
\end{aligned}$$

where Equation (3.28) holds because ε_i are identically distributed random variables, for $i = 1, 2, \ldots, m$.

Case 2. $Q_d \in I_3$.

Since $Q_d \in a_{k_1}$, the MCP with hedge price and hedge quantity is

$$f_{I_3}(Q_d) = P^s_{k_1}(1 + \varepsilon_{k_1}). \qquad (3.29)$$

Thus, the variance of MCP with hedge price and hedge quantity is

$$Var(f_{I_3}(Q_d)) = Var(P^s_{k_1}(1 + \varepsilon_{k_1})) = {P^s_{k_1}}^2\sigma^2. \qquad (3.30)$$

We now show that the variance of MCP does not decrease for $\rho_{\varepsilon_i\varepsilon_j} \in [0, 1]$ when the hedge quantity is introduced.

Theorem 3.5. *For $\rho_{\varepsilon_i\varepsilon_j} \in [0,1]$, we have $Var(\hat{P}_s) \geq Var(P_s)$.*

Proof. From Assumption 3-2, $Q_d \in (\bar{Q}, \hat{Q}]$. There are two cases: $Q_d \in I_2$ and $Q_d \in I_3$.

Case 1. $Q_d \in I_2$.

For $\rho_{\varepsilon_i\varepsilon_j} \in [0,1]$, we have

$$\begin{aligned}
Var(f_{I_2}(Q_d)) &= \left({P^s_{k_1}}^2 + \frac{\sum_{i=1}^{k_0-1} A_i^2 P_i^{s2} + (\bar{Q} - Q^s_{k_0-1})^2 {P^s_{k_0}}^2}{L^2} \right) \sigma^2 \\
&\quad +2 \sum_{\substack{i=1,2,\ldots,k_0-1,\\ j=1,2,\ldots,k_0-1:\\ i<j}} \frac{A_i P^s_i A_j P^s_j \rho_{\varepsilon_i\varepsilon_j} \sigma^2}{L^2} \\
&\quad +2 \sum_{i=1,2,\ldots,k_0-1} \frac{A_i P^s_i (\bar{Q} - Q^s_{k_0-1}) P^s_{k_0-1} \rho_{\varepsilon_i\varepsilon_{k_0}} \sigma^2}{L^2} \\
&\quad +2 \sum_{i=1,2,\ldots,k_0-1} \frac{A_i P^s_i P^s_{k_1} \rho_{\varepsilon_i\varepsilon_{k_1}} \sigma^2}{L} \\
&\quad +2 \frac{P^s_{k_1} (\bar{Q} - Q^s_{k_0-1}) P^s_{k_0} \rho_{\varepsilon_{k_0}\varepsilon_{k_1}} \sigma^2}{L} \\
&> {P^s_{k_1}}^2 \sigma^2 \qquad (3.31) \\
&= Var(P_s),
\end{aligned}$$

where Inequality (3.31) holds from Equations (3.25) and (3.28). Note that $\sum_{i=1}^{k_0-1} A_i^2 P_i^{s2}$ $+(\bar{Q} - Q^s_{k_0-1})^2 {P^s_{k_0}}^2 > 0$ because $\bar{Q} > Q^s_{k_0-1}$.

Case 2. $Q_d \in I_3$.

We have

$$Var(f_{I_3}(Q_d)) = {P^s_{k_1}}^2 \sigma^2 = Var(P_s), \qquad (3.32)$$

where Equation (3.32) holds from Equations (3.25) and (3.30).

From cases 1 and 2, $\hat{P}_s = f(Q_d)$, we have $Var(\hat{P}_s) \geq Var(P_s)$. □

Next, we show other factors that affect the variance of the MCP.

Theorem 3.6. *For $Q_d \in I_2$ and $\rho_{\varepsilon_i\varepsilon_j} \in [0,1]$, $Var(\hat{P}_s)$ is a decreasing function of L, an increasing function of $\bar{Q}$. For $Q_d \in I_2$, $Var(\hat{P}_s)$ is also independent of $\bar{P}$. Moreover, for $Q_d \in I_3$, $Var(\hat{P}_s)$ is independent of L, $\bar{Q}$ and $\bar{P}$.*

Proof. Similar to Theorem 3.2. □

3.3.4 CP

When no hedge price and hedge quantity are given, customers buy all electricity from the wholesale market at MCP. Thus, the CP without hedge price and hedge quantity is $P_s^c = P_s$.

When hedge price and hedge quantity exist, customers pay the hedge price, $\bar{P}$, for the hedge quantity, $\bar{Q}$. To meet the demand, customers buy the remaining $Q_d - \bar{Q}$ units of electricity from the wholesale market with MCP, $\hat{P}_s$. We define the CP as

$$\begin{aligned} \hat{P}_s^c &= \omega\bar{P} + (1-\omega)\hat{P}_s \\ &= \begin{cases} \omega\bar{P} + (1-\omega)f_{I_2}(Q_d), & \text{for } Q_d \in I_2, \\ \omega\bar{P} + (1-\omega)f_{I_3}(Q_d), & \text{for } Q_d \in I_3, \end{cases} \end{aligned} \tag{3.33}$$

where $\omega = \bar{Q}/Q_d$ is the hedge ratio and Equation (3.33) holds from Equation (3.22) for $Q_d \in (\bar{Q}, \hat{Q}]$.

We now consider two cases: $Q_d \in I_2$ and $Q_d \in I_3$.

Case 1. $Q_d \in I_2$.

The CP with hedge price and hedge quantity is

$$\begin{aligned} \hat{P}_s^c = \omega\bar{P} + (1-\omega)\Big(&P_{k_1}^s(1+\varepsilon_{k_1}) \\ &-\frac{\bar{P}\bar{Q} - \left[\sum_{i=1}^{k_0-1} A_i P_i^s + (\bar{Q} - Q_{k_0-1}^s)P_{k_0}^s\right]}{L} \\ &+\frac{\sum_{i=1}^{k_0-1} A_i P_i^s \varepsilon_i + (\bar{Q} - Q_{k_0-1}^s)P_{k_0}^s \varepsilon_{k_0}}{L}\Big), \end{aligned} \tag{3.34}$$

where Equation (3.34) holds from Equations (3.27) and (3.33). Consequently, the variance of CP with hedge price and hedge quantity is

$$\begin{aligned}
Var(\hat{P}_s^c) &= Var\Big[\omega\bar{P}+(1-\omega)\Big(P_{k_1}^s(1+\varepsilon_{k_1}) \\
&\qquad -\frac{\bar{P}\bar{Q}-\left[\sum_{i=1}^{k_0-1}A_iP_i^s+(\bar{Q}-Q_{k_0-1}^s)P_{k_0}^s\right]}{L} \\
&\qquad +\frac{\sum_{i=1}^{k_0-1}A_iP_i^s\varepsilon_i+(\bar{Q}-Q_{k_0-1}^s)P_{k_0}^s\varepsilon_{k_0}}{L}\Big)\Big] \\
&= (1-\omega)^2\Bigg[\left({P_{k_1}^s}^2+\frac{\sum_{i=1}^{k_0-1}A_i^2{P_i^s}^2+(\bar{Q}-Q_{k_0-1}^s)^2{P_{k_0}^s}^2}{L^2}\right)\sigma^2 \\
&\qquad +2\sum_{\substack{i=1,2,\ldots,k_0-1,\\ j=1,2,\ldots,k_0-1:\\ i<j}}\frac{A_iP_i^sA_jP_j^s\rho_{\varepsilon_i\varepsilon_j}\sigma^2}{L^2} \\
&\qquad +2\sum_{i=1,2,\ldots,k_0-1}\frac{A_iP_i^s(\bar{Q}-Q_{k_0-1}^s)P_{k_0-1}^s\rho_{\varepsilon_i\varepsilon_{k_0}}\sigma^2}{L^2} \\
&\qquad +2\sum_{i=1,2,\ldots,k_0-1}\frac{A_iP_i^sP_{k_1}^s\rho_{\varepsilon_i\varepsilon_{k_1}}\sigma^2}{L} \\
&\qquad +2\frac{P_{k_1}^s(\bar{Q}-Q_{k_0-1}^s)P_{k_0}^s\rho_{\varepsilon_{k_0}\varepsilon_{k_1}}\sigma^2}{L}\Bigg],
\end{aligned} \tag{3.35}$$

where Equation (3.35) holds because ε_i are identically distributed random variables, for $i = 1, 2, \ldots, m$.

Case 2. $Q_d \in I_3$.

The CP with hedge price and hedge quantity is

$$\hat{P}_s^c = \omega\bar{P} + (1-\omega)[P_{k_1}^s(1+\varepsilon_{k_1})], \tag{3.36}$$

where Equation (3.36) holds from Equations (3.29) and (3.33). Thus, the variance of CP with hedge price and hedge quantity is

$$Var(\hat{P}_s^c) = (1-\omega)^2{P_{k_1}^s}^2\sigma^2. \tag{3.37}$$

We now show that the variance of CP does not increase when hedge quantity is assigned.

Lemma 3.2. *For* $Q_d \in I_2$,

$$(1-\omega)^2\left(P^s_{k_1}+\frac{\sum_{i=1}^{k_0-1}A_iP^s_i+(\bar{Q}-Q^s_{k_0-1})P^s_{k_0}}{L}\right)^2 \le {P^s_{k_1}}^2.$$

Proof. For $Q_d \in I_2$, we have

$$\frac{\bar{Q}}{\bar{Q}+L} \le \frac{\bar{Q}}{Q_d} < 1,$$

where the inequality holds from $\bar{Q} < Q_d \le \bar{Q}+L$. Thus,

$$\begin{aligned} 0 &< 1-\frac{\bar{Q}}{Q_d} \le 1-\frac{\bar{Q}}{\bar{Q}+L}, \qquad \text{and} \\ 0 &< \left(1-\frac{\bar{Q}}{Q_d}\right)^2 \le \left(\frac{L}{\bar{Q}+L}\right)^2. \end{aligned} \tag{3.38}$$

Hence,

$$\begin{aligned} &(1-\omega)^2\left(P^s_{k_1}+\frac{\sum_{i=1}^{k_0-1}A_iP^s_i+(\bar{Q}-Q^s_{k_0-1})P^s_{k_0}}{L}\right)^2 \\ &= \left(1-\frac{\bar{Q}}{Q_d}\right)^2\left(P^s_{k_1}+\frac{\sum_{i=1}^{k_0-1}A_iP^s_i+(\bar{Q}-\sum_{i=1}^{k_0-1}A_i)P^s_{k_0}}{L}\right)^2 \\ &< \left(1-\frac{\bar{Q}}{Q_d}\right)^2\left(P^s_{k_1}+\frac{\bar{Q}P^s_{k_0}}{L}\right)^2 &(3.39) \\ &\le \left(\frac{L}{\bar{Q}+L}\right)^2\left(P^s_{k_1}+\frac{\bar{Q}P^s_{k_0}}{L}\right)^2 &(3.40) \\ &= \left(\frac{P^s_{k_1}L+\bar{Q}P^s_{k_0}}{\bar{Q}+L}\right)^2 \\ &< \left(\frac{L+\bar{Q}}{\bar{Q}+L}\right)^2{P^s_{k_1}}^2 &(3.41) \\ &= {P^s_{k_1}}^2, \end{aligned}$$

where Inequalities (3.39) and (3.41) hold from $P^s_1 < P^s_2 < \cdots < P^s_m$ and Inequality (3.40) holds from Equation (3.38). □

Theorem 3.7. $Var(\hat{P}^c_s) \le Var(P^c_s)$.

Proof. Consider two cases: $Q_d \in I_2$ and $Q_d \in I_3$.

Case 1. $Q_d \in I_2$.

For $\rho_{\varepsilon_i \varepsilon_j} = 1$, we have

$$\begin{aligned} Var(\hat{P}_s^c) &= (1-\omega)^2 \times \left(P_{k_1}^s + \frac{\sum_{i=1}^{k_0-1} A_i P_i^s + (\bar{Q} - Q_{k_0-1}^s) P_{k_0}^s}{L} \right)^2 \sigma^2 \\ &\leq {P_{k_1}^s}^2 \sigma^2 \qquad (3.42) \\ &= Var(P_s) \\ &= Var(P_s^c), \end{aligned}$$

where Equation (3.42) holds from Lemma 3.2, Equations (3.25) and (3.35). Since $Var(\hat{P}_s^c)$ is an increasing function of $\rho_{\varepsilon_i \varepsilon_j}$ and $Var(\hat{P}_s^c) \leq Var(P_s^c)$ for $\rho_{\varepsilon_i \varepsilon_j} = 1$, we have $Var(\hat{P}_s^c) \leq Var(P_s^c)$ for all possible values of $\rho_{\varepsilon_i \varepsilon_j} \in [-1, 1]$.

Case 2. $Q_d \in I_3$.

Since $0 < \omega \leq 1$,

$$\begin{aligned} Var(\hat{P}_s^c) &= (1-\omega)^2 {P_{k_1}^s}^2 \sigma^2 \\ &< {P_{k_1}^s}^2 \sigma^2 \\ &= Var(P_s) \\ &= Var(P_s^c), \end{aligned}$$

where the inequality holds from Equations (3.25) and (3.37). □

Next, we show other factors that affect the variance of CP.

Theorem 3.8. *For $Q_d \in I_2$ and $\rho_{\varepsilon_i \varepsilon_j} \in [0, 1]$, $Var(\hat{P}_s^c)$ is a decreasing function of L. For $Q_d \in I_2$, $Var(\hat{P}_s^c)$ is independent of $\bar{P}$. Moreover, for $Q_d \in I_3$, $Var(\hat{P}_s^c)$ is independent of L and $\bar{P}$.*

Proof. Similar to Theorem 3.2. □

3.4 Numerical study

In Singapore, the Uniform Singapore Energy Price (USEP) is the MCP decided by the electricity wholesale market. In 2004, vesting contracts are introduced in the Singapore electricity market. They are a kind of bilateral contract, except that they are imposed by the Singapore government and are not negotiated. The two basic elements of vesting contracts are hedge price and hedge quantity. The percentage of total demand hedged by contracts is the hedge ratio, which is the hedge quantity divided by demand. The CP is the combination of hedge price and MCP weighted by the hedge ratio. The data of hedge price, hedge quantity and hedge ratio can be found in the website of the Market Support Services Licensee. The data of USEP and demand quantity can be found in the website of Energy Market Company.

We analyze the data collected from 2003 to 2010. In the Singapore wholesale market, one trading period is half an hour. From 2003 to 2010, the number of periods is 140,256. The hedging mechanism was not part of any 2003 market data. Also, different hedge prices and hedge quantities are assigned from 2004 to 2010.

To avoid irrational prices, we only select these periods with USEP within [0, S$1000/MWh] for analysis. Irrational prices are usually due to unplanned disruption of gas. In fact, 99.90% of the total periods has USEPs within the interval [0, S$1000/MWh]. In Table 3.2, we present the number of selected periods and the total number of periods. Also, the percentage of selected periods for each year, which is at least 99.72%, is showed in Table 3.2.

Table 3.2: The number of selected periods with USEP in [0, S$1000/MWh]

Year	Selected periods	Total periods	Percentage (Selected/Total)
2003	17499	17520	99.88%
2004	17550	17568	99.90%
2005	17503	17520	99.90%
2006	17508	17520	99.93%
2007	17508	17520	99.93%
2008	17557	17568	99.94%
2009	17471	17520	99.72%
2010	17517	17520	99.98%
Total	140113	140256	99.90%

Table 3.3: Mean and sample variance of USEP for different demand intervals (Based on percentiles of demand)

Year	Demand interval	Mean of USEP	Sample variance of USEP
2003	$[2535, 3158]$	70	205
	$(3158, 3305]$	77	544
	$(3305, 3462]$	80	89
	$(3462, 3608]$	84	363
	$(3608, 3793]$	86	262
	$(3793, 4082]$	89	477
	$(4082, 4288]$	93	402
	$(4288, 4529]$	97	551
	$(4529, 4644]$	107	912
	$(4644, 4999]$	117	1385
2004	$[2691, 3308]$	67	99
	$(3308, 3473]$	74	94
	$(3473, 3637]$	78	413
	$(3637, 3784]$	83	1427
	$(3784, 3972]$	81	572
	$(3972, 4245]$	81	134
	$(4245, 4470]$	83	379
	$(4470, 4682]$	82	230
	$(4682, 4816]$	87	1409
	$(4816, 5091]$	92	945
2005	$[2879, 3496]$	84	230
	$(3496, 3640]$	96	288
	$(3640, 3799]$	109	1697
	$(3799, 3953]$	110	2747
	$(3953, 4146]$	106	1273
	$(4146, 4400]$	108	1292
	$(4400, 4648]$	113	2962
	$(4648, 4868]$	107	2668
	$(4868, 5027]$	111	1477
	$(5027, 5359]$	135	6839
2006	$[3021, 3600]$	105	256
	$(3600, 3753]$	115	647
	$(3753, 3920]$	124	877
	$(3920, 4091]$	129	2164
	$(4091, 4279]$	130	2913
	$(4279, 4556]$	135	5051
	$(4556, 4815]$	147	7525
	$(4815, 5030]$	130	2919
	$(5030, 5203]$	137	2689
	$(5203, 5452]$	153	4769
2007	$[3208, 3754]$	104	307
	$(3754, 3919]$	111	217
	$(3919, 4108]$	116	471
	$(4108, 4292]$	122	1713
	$(4292, 4487]$	124	1774
	$(4487, 4763]$	131	4893
	$(4763, 5024]$	126	945
	$(5024, 5223]$	125	763
	$(5223, 5375]$	130	619
	$(5375, 5782]$	140	3041
2008	$[3082, 3753]$	118	2009
	$(3753, 3941]$	151	2032

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Table 3.3: Mean and sample variance of USEP for different demand intervals (Based on percentiles of demand) – Continued

Year	Demand interval	Mean of USEP	Sample variance of USEP
	$(3941, 4130]$	156	1534
	$(4130, 4323]$	160	1997
	$(4323, 4531]$	163	1997
	$(4531, 4801]$	164	3098
	$(4801, 5084]$	168	4689
	$(5084, 5280]$	161	2997
	$(5280, 5466]$	176	2456
	$(5466, 5949]$	201	3453
2009	$[3053, 3758]$	82	609
	$(3758, 3956]$	114	796
	$(3956, 4139]$	123	1724
	$(4139, 4345]$	132	2836
	$(4345, 4556]$	140	4405
	$(4556, 4805]$	140	5854
	$(4805, 5093]$	139	4982
	$(5093, 5289]$	141	5139
	$(5289, 5528]$	180	10149
	$(5528, 5876]$	239	15351
2010	$[3577, 4154]$	132	172
	$(4154, 4327]$	142	559
	$(4327, 4541]$	155	1463
	$(4541, 4754]$	155	1260
	$(4754, 4961]$	160	1729
	$(4961, 5231]$	172	3462
	$(5231, 5506]$	172	2644
	$(5506, 5721]$	180	4120
	$(5721, 5899]$	196	5831
	$(5899, 6294]$	232	12496

Table 3.4: Correlation coefficient between sample variance and square of mean of USEP for selected periods (Based on percentiles of demand)

Year	2003	2004	2005	2006	2007	2008	2009	2010
Correlation coefficient	0.90	0.65	0.93	0.86	0.67	0.48	0.97	0.98

Table 3.5: Mean and sample variance of USEP for different demand intervals (Based on the same length of demand range)

Year	Demand interval	Mean of USEP	Sample variance of USEP
2003	$[2535, 2781]$	60	552
	$(2781, 3027]$	67	177
	$(3027, 3274]$	75	445
	$(3274, 3520]$	80	94
	$(3520, 3767]$	85	369
	$(3767, 4013]$	88	455
	$(4013, 4260]$	92	353
	$(4260, 4506]$	97	617
	$(4506, 4752]$	110	1102
	$(4752, 4999]$	118	1247
2004	$[2691, 2931]$	45	628
	$(2931, 3171]$	64	79
	$(3171, 3411]$	71	66
	$(3411, 3651]$	78	470
	$(3651, 3891]$	82	871
	$(3891, 4131]$	82	513
	$(4131, 4371]$	82	334
	$(4371, 4611]$	83	288
	$(4611, 4851]$	86	1042
	$(4851, 5091]$	93	994
2005	$[2879, 3127]$	67	147
	$(3127, 3375]$	80	188
	$(3375, 3623]$	94	252
	$(3623, 3871]$	108	1470
	$(3871, 4119]$	108	2343
	$(4119, 4467]$	108	1226
	$(4367, 4615]$	113	2781
	$(4615, 4863]$	107	2764
	$(4863, 5111]$	114	2225
	$(5111, 5359]$	155	10594
2006	$[3021, 3264]$	100	67
	$(3264, 3507]$	103	266
	$(3507, 3750]$	113	540
	$(3750, 3993]$	126	1273
	$(3993, 4236]$	130	2667
	$(4236, 4479]$	134	5099
	$(4479, 4722]$	142	4843
	$(4722, 4966]$	139	6197
	$(4966, 5209]$	135	2343
	$(5209, 5452]$	153	4780
2007	$[3208, 3465]$	90	59
	$(3465, 3723]$	105	317
	$(3723, 3980]$	111	242
	$(3980, 4237]$	120	1085
	$(4237, 4495]$	124	1785
	$(4495, 4752]$	132	5158
	$(4752, 5010]$	126	1027
	$(5010, 5267]$	126	740
	$(5267, 5525]$	132	1242
	$(5525, 5782]$	155	6378
2008	$[3082, 3368]$	61	680
	$(3368, 3655]$	107	1660

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Table 3.5: Mean and sample variance of USEP for different demand intervals (Based on the same length of demand range)– Continued

Year	Demand interval	Mean of USEP	Sample variance of USEP
	(3655, 3942]	147	1957
	(3942, 4229]	157	1626
	(4229, 4515]	163	1962
	(4515, 4802]	165	3025
	(4802, 5089]	168	4670
	(5089, 5375]	162	2631
	(5375, 5662]	192	2750
	(5662, 5949]	225	7557
2009	[3053, 3335]	62	160
	(3335, 3617]	73	302
	(3617, 3900]	107	688
	(3900, 4182]	123	1632
	(4182, 4465]	135	3520
	(4465, 4747]	141	5505
	(4747, 5029]	139	4884
	(5029, 5312]	143	5383
	(5312, 5594]	185	9362
	(5594, 5876]	263	18431
2010	[3577, 3849]	133	44
	(3849, 4121]	132	168
	(4121, 4392]	143	585
	(4392, 4664]	157	1543
	(4664, 4936]	157	1373
	(4936, 5207]	171	3563
	(5207, 5479]	177	2897
	(5479, 5751]	179	3663
	(5751, 6022]	211	9163
	(5022, 6294]	244	12402

Table 3.6: Correlation coefficient between sample variance and square of mean of USEP for selected periods (Based on the same length of demand range)

Year	2003	2004	2005	2006	2007	2008	2009	2010
Correlation coefficient	0.81	0.46	0.95	0.81	0.82	0.84	0.99	0.99

In this section, we first investigate the Singapore electricity market to see whether it follows analytical model 1 or analytical model 2, that is, whether uncertainty of supply is an additive or multiplicative factor. Secondly, we estimate USEP for certain selected demand intervals. Finally, we verify the main results in this chapter, that is, we test if the coefficient of variation of CP is reduced with hedge. Also, the relationship between coefficient of variation of CP and hedge price is examined.

3.4.1 Uncertainty is an additive or multiplicative factor

In this part, we examine the data to see if the sample variance of USEP follows the assumption of analytical model 1 or analytical model 2. From Assumption 3-1, the supply function in analytical model 1 is

$$F(x) = P_i^s + \varepsilon_i, \quad \text{for} \;\; x \in a_i,$$

From the above equation, the variance of MCP is σ^2. From Assumption 3-1$'$, the supply function in analytical model 2 is

$$F(x) = P_i^s(1 + \varepsilon_i), \quad \text{for} \;\; x \in a_i,$$

From the above equation, the variance of MCP is ${P_i^s}^2\sigma^2$, for $x \in a_i$.

We now test sample variance of USEP to see whether it follows analytical model 1 or analytical model 2. We partition the selected data of each year into 10 groups of equal size according to percentiles of demand. The mean of USEP and sample variance of USEP for each demand interval are presented in Table 3.3. The correlation coefficient between sample variance and square of mean of USEP for each year is presented in Table 3.4. We observe that correlation coefficients for all 8 years are positive. Moreover, the correlation coefficients for 5 years are greater than 0.8.

To further example the impact of the length of demand range. We partition the selected data of each year into 10 equal size groups according to length of demand. The mean of USEP and sample variance of USEP for each demand interval are presented in Table 3.5. The correlation coefficient between sample variance and square of mean of USEP for each year is presented in Table 3.6. We observe that correlation coefficients for all 8 years are positive. Moreover, the correlation coefficients for 7 years are greater than 0.8. Hence, sample variance of USEP has highly positive correlation with square of mean of USEP. As a result, there is strong evidence to suggest that uncertainty is a multiplicative factor. That is, analytical model 2 should be applied to the Singapore electricity market.

Table 3.7: Minimum and maximum demands for selected periods

Year	2003	2004	2005	2006	2007	2008	2009	2010
Minimum demand (MW)	2535	2691	2879	3021	3208	3082	3053	3577
Maximum demand (MW)	4999	5091	5359	5452	5782	5949	5876	6294

Table 3.8: Estimated USEP of selected demand intervals

Demand interval	Estimated USEP
$[2500, 3000]$	63
$(3000, 3500]$	78
$(3500, 4000]$	107
$(4000, 4500]$	124
$(4500, 5000]$	127
$(5000, 5500]$	151
$(5500, 6000]$	203
$(6000, 6500]$	240

3.4.2 Estimation of USEP for certain selected demand intervals

In this part, we estimate USEPs for certain selected demand intervals. In Table 3.7, we present the minimum and maximum demands of selected periods from 2003 to 2010. The minimum and maximum demands of all years are 2535MW and 6294MW, respectively. Thus, we choose the demand range [2500, 6500] and partition it into 8 intervals. The demand intervals selected are [2500, 3000], (3000, 3500], (3500, 4000], (4000, 4500], (4500, 5000], (5000, 5500], (5500, 6000] and (6000, 6500].

The selected data of each year are partitioned into 10 groups of equal size based on percentiles of demand. In the second and third columns of Table 3.3, the mean of USEP for each demand interval is presented. They are also shown in Figure 3.2. From these two columns, we now estimate the USEPs for 8 selected demand intervals: [2500, 3000], (3000, 3500], (3500, 4000], (4000, 4500], (4500, 5000], (5000, 5500], (5500, 6000] and (6000, 6500]. The estimated USEP for each selected demand interval is the average of all possible means of USEPs in that interval. They are listed in Table 3.8 and shown in Figure 3.3. We observe that the USEP increases as the demand increases.

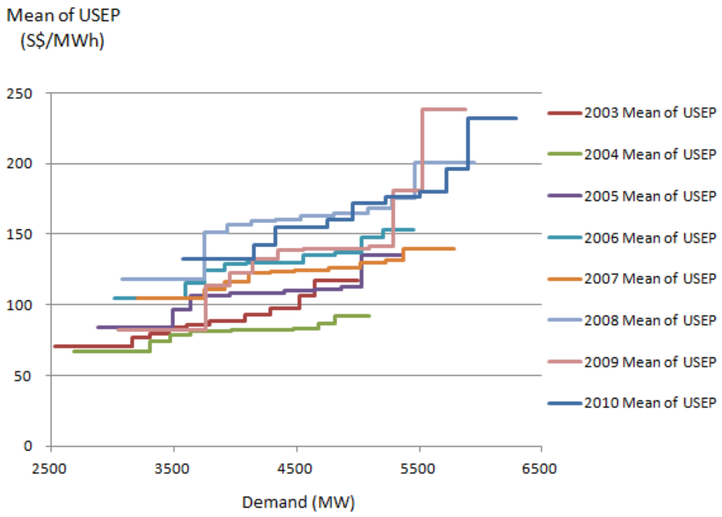

Figure 3.2: Mean of USEP for demand intervals

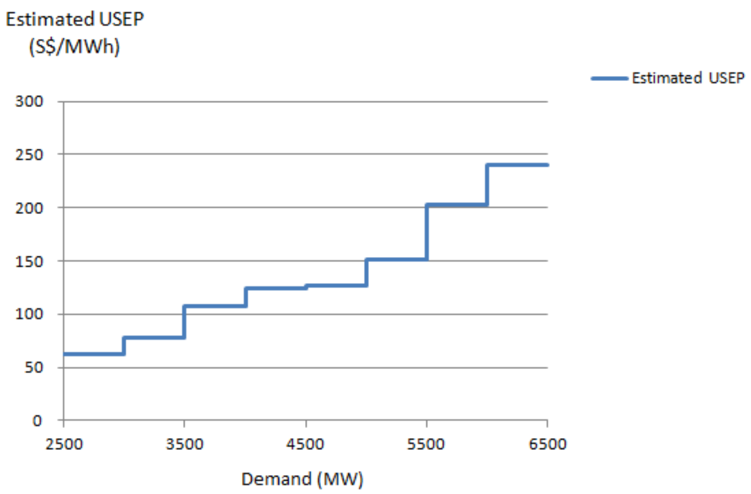

Figure 3.3: Estimated USEP for selected demand intervals

Table 3.9: Coefficient of variation of customer price (CP) for selected periods

Year	Quarter	Hedge price ($/MWh)	Hedge ratio	Coefficient of variation of CP
2003	Q1	94.24	0.00	0.28
	Q2	96.25	0.00	0.21
	Q3	95.73	0.00	0.24
	Q4	101.56	0.00	0.39
2004	Q1	94.24	0.65	0.05
	Q2	96.25	0.65	0.07
	Q3	95.73	0.65	0.04
	Q4	101.56	0.65	0.15
2005	Q1	101.29	0.65	0.05
	Q2	96.35	0.65	0.07
	Q3	117.38	0.65	0.04
	Q4	128.39	0.65	0.15
2006	Q1	140.70	0.65	0.09
	Q2	139.44	0.65	0.14
	Q3	147.90	0.65	0.17
	Q4	150.20	0.65	0.14
2007	Q1	134.66	0.65	0.18
	Q2	121.14	0.65	0.08
	Q3	137.25	0.55	0.10
	Q4	150.04	0.55	0.05
2008	Q1	161.80	0.55	0.13
	Q2	174.44	0.55	0.13
	Q3	183.25	0.55	0.07
	Q4	238.64	0.55	0.14
2009	Q1	167.14	0.55	0.15
	Q2	115.26	0.55	0.33
	Q3	138.92	0.55	0.19
	Q4	161.70	0.55	0.18
2010	Q1	171.05	0.55	0.21
	Q2	176.10	0.55	0.13
	Q3	176.29	0.55	0.09
	Q4	165.71	0.55	0.15

Table 3.10: Mean of coefficients of variations of customer price (CP) for different hedge ratios

Hedge ratio	0	55%	65%
Mean of coefficients of variations of CP	0.28	0.15	0.10

3.4.3 Coefficient of variation of CP and hedge ratio

The coefficients of variations of CP in different quarters are presented in Table 3.9. The means for coefficients of variations of CP in different quarters are presented in Table 3.10. From Table 3.10, the mean for coefficients of variations of CP with no hedge is 0.28. Moreover, the means for coefficients of variations of CP with 55% and 65% hedge ratios are 0.15 and 0.10, respectively. Hence, the coefficient of variation of CP is reduced with hedge. This result supports Theorems 3.3 and 3.7.

To further verify Theorems 3.3 and 3.7, we conduct two tests: Test 3.1 and Test 3.2. In Test 3.1, we verify the relationship between coefficients of variations of CP with no hedge ratio and with 65% hedge ratio. In Test 3.2, we verify the relationship between coefficients of variations of CP with no hedge ratio and with 55% hedge ratio.

Test 3.1: Coefficient of variation of CP with 65% hedge ratio is less than coefficient of variation of CP with no hedge

There is no hedge in 2003. The quarterly coefficients of variations of USEP for 2003 are presented in Table 3.9. From 2004 to Quarter 2, 2007, the hedge ratio is 65%. Also, the quarterly coefficients of variations of USEP from 2004 to Quarter 2, 2007 are presented in Table 3.9.

Now, we test if the coefficient of variation of CP with 65% hedge ratio is less than the coefficient of variation of CP with no hedge. The null hypothesis H_0 is that the mean for coefficients of variations of CP with 65% hedge ratio is equal to the mean for coefficients of variations of CP with no hedge. That is

$$\begin{aligned} H_0 : \bar{X}_H &= \bar{X}, \\ H_1 : \bar{X}_H &< \bar{X}, \end{aligned}$$

where $\bar{X}$ is the mean for coefficients of variations of CP with no hedge and $\bar{X}_H$ is the mean for coefficients of variations of CP with 65% hedge ratio.

In this test, we set the significance level at 0.05. The probability of Type I error is 0.02, which is less than the significance level. Thus, we reject H_0. Statistically, mean for coefficients of variations of CP with 65% hedge ratio is significantly less than mean for coefficients of variations of CP with no hedge. This result supports Theorems 3.3 and 3.7.

Test 3.2: Coefficient of variation of CP with 55% hedge ratio is less than coefficient of variation of CP with no hedge

There is no hedge in 2003. The quarterly coefficients of variation of USEP for 2003 are presented in Table 3.9. From Quarter 3, 2007 to 2010, the hedge ratio is 55%. Also, the quarterly coefficients of variation of USEP from Quarter 3, 2007 to 2010 are presented in Table 3.9.

Now, we test if the coefficient of variation of CP with 55% hedge ratio is less than the coefficient of variation of CP with no hedge. The null hypothesis H_0 is that the mean for coefficients of variations of CP with 55% hedge ratio is equal to the mean for coefficients of variations of CP with no hedge. That is

$$\begin{aligned} H_0 : \bar{X}_L &= \bar{X}, \\ H_1 : \bar{X}_L &< \bar{X}, \end{aligned}$$

where $\bar{X}$ is the mean for coefficients of variations of CP with no hedge and $\bar{X}_L$ is the mean for coefficients of variations of CP with 55% hedge ratio.

In this test, we set the significance level at 0.05. The probability of Type I error is 0.01, which is less than the significance level. Thus, we reject H_0. Statistically, the mean for coefficients of variations of CP with 55% hedge ratio is significantly less than the mean for coefficients of variations of CP with no hedge. This result supports Theorems 3.3 and 3.7.

3.4.4 Coefficient of variation of CP and hedge price

The coefficients of variations of CP and hedge price in different quarters are presented in Table 3.9. We now exam the relationship between coefficient of variation of CP and hedge price. The correlation coefficient between coefficient of variation of CP and hedge price is -0.11. This result supports Theorems 3.4 and 3.8.

3.5 Concluding remarks

In this chapter, we consider an unstable environment and assume that supply is a discrete function. To study the impact of a hedge, we build mathematical models and analyze how the hedge price and quantity affect the uncertainties of MCP and CP.

There are four major analytical results from our models. Firstly, we find that the variance of MCP increases when the hedge quantity is assigned. That is because the uncertainty (risk) of supplying an amount of electricity equal to hedge quantity is shifted to the estimated demand beyond the hedge quantity. Also, the variance of CP decreases when the hedge quantity is assigned.

Secondly, we find that the variances of MCP and CP do not have statistically significant relationships with the hedge price. That is, the variances of MCP and CP are affected by the uncertainty of profit R and the quantity L after neutralization. Also, the uncertainty of R is caused by the supply function. Thus, the hedge price may affect only the total amount of R, and not its variance.

Thirdly, we find that the variances of MCP and CP are decreasing functions of neutralizing quantity L. That is because a larger L can help to share the uncertainty for supplying the hedge quantity.

Fourthly, we find that the variance of MCP is an increasing function of hedge quantity $\bar{Q}$. That is because large $\bar{Q}$ results in large uncertainty being shifted to the neutralized interval I_2.

A numerical study is conducted using data from the Singapore electricity market from 2003 to 2010 to verify our model assumptions and the main results. The data are also used to conduct parameter estimation.

Chapter 4

Competition Markets and Bilateral Contracts

4.1 Introduction

Since the 1980s, the electricity industries in many countries have been deregulated, for instance, in United States, Australia, United Kingdom, New Zealand, Norway and Spain (Joskow, 2008). As a result of deregulation, the electricity spot markets are introduced. There are three participants in the electricity spot markets: suppliers, customers and ISO. Usually, the suppliers are gencos who bid to sell electricity to customers. Then, the ISO collects information from both supply and demand sides and dispatches electricity for each period. This market mechanism may result in an undesirable outcome, the volatility of electricity prices.

To reduce the high volatility of electricity prices, vesting and forward contracts are introduced into the spot markets. Before submitting bids in the spot market, gencos and customers may sign contracts on dispatching an amount of electricity (contract quantity) at a fixed price (contract price). The bilateral contracts are independent from the market dispatching mechanism. They are just financial instruments without any actual transfer of electricity.

In this chapter, we consider two different models, SFE and Cournot, to study the interactions between bilateral contracts and competition behaviors of gencos in the spot market. The SFE model was first proposed by Klemperer and Meyer (1989). In the SFE model, each genco determines its own supply function and its goal is to maximize individual profit.

Linear and symmetric SFE models are both popular SFE models. In the linear SFE models, each genco determines a linear supply function. For the symmetric SFE models, all gencos have the same cost function. With demand uncertainty, Klemperer and Meyer (1989) assumed that symmetric gencos submit general supply functions and proved the existence of SFE. They also gave sufficient conditions for the uniqueness of SFE. Baldick et al. (2004) presented a linear SFE model and applied it to the United Kingdom electricity market. Rudkevich (2005) proved the existence and uniqueness of SFE in linear models. Moreover, many researchers examine the effects of bilateral contracts on SFE. For example, Niu et al. (2005) proposed a linear asymmetric SFE model with transmission constraints to study the behaviors of gencos with bilateral contracts.

Another popular model is the Cournot model. The Cournot model is a well known oligopoly model studied by many researchers. In the Cournot model, gencos determine only the quantities that they are willing to supply. Willems (2002) studied two-genco Cournot models without bilateral contracts. With bilateral contracts, Allaz and Vila (1993) presented a two-genco Cournot model to investigate the impact of bilateral contracts on the market prices. Bushnell (2007) generalized this model with multiple gencos.

The remaining parts of this chapter are organized as follows. In Section 4.2, we present two models, SFE and Cournot, to investigate the impact of bilateral contracts on the variances of MCP and CP. In Section 4.3, a numerical study based on Singapore electricity market is conducted to verify our models. Conclusions are presented in Section 4.4.

4.2 The SFE and Cournot models

In this section, we consider a spot market where n gencos are in competition to supply electricity and bilateral contracts are signed outside the market. A part of demand is satisfied in the bilateral contracts and the other part is satisfied in the spot market.

We present two models, SFE and Cournot, to investigate the impact of bilateral contracts on the variances of MCP and CP in the spot market. In the SFE model, each genco determines its own supply function. However, in the Cournot model, each genco only determines the quantity it is willing to supply. In both models, the goal of each genco is to maximize its own profit in the spot market. We assume that market demand function is linear with uncertainty in both models. Also, we assume that the production cost is quadratic and marginal cost is linear.

4.2.1 Notations

The following notations are used in this chapter.

Table 4.1: Notations used in the SFE and Cournot models

n	=	Number of generation companies (gencos)
i	=	Genco i, for $i = 1, 2, \ldots, n$
x_i	=	Electricity quantity produced by genco i, for $i = 1, 2, \ldots, n$, where $x_i \geq 0$
x	=	Total electricity quantity produced, where $x = \sum_{i=1}^{n} x_i$
$P_C(x_i)$	=	Production cost function for genco i, for $i = 1, 2, \ldots, n$
$M_C(x_i)$	=	Marginal cost function for genco i, for $i = 1, 2, \ldots, n$
p	=	Trading price, where $p \geq 0$
$Q(p)$	=	Demand function
$S_i(p)$	=	Supply function for genco i, for $i = 1, 2, \ldots, n$, where $S_i(p) \geq 0$
$S_{-i}(p)$	=	Aggregated supply function for gencos other than genco i, where $S_{-i}(p) = \sum_{i=1}^{n} S_i(p) - S_i(p)$
$\bar{P}$	=	Contract price, where $\bar{P} \geq 0$
$\bar{Q}_i$	=	Contract quantity for genco i, for $i = 1, 2, \ldots, n$, where $0 < \bar{Q}_i \leq S_i(p)$
$\bar{Q}$	=	Total contract quantity, where $\bar{Q} = \sum_{i=1}^{n} \bar{Q}_i$
$\hat{Q}_s$	=	Market clearing quantity with contracts, where $\hat{Q}_s \geq \bar{Q} \geq 0$
ω	=	Hedge ratio, where $\omega \in (0, 1]$
	=	$\bar{Q}/\hat{Q}_s$
$\hat{P}_s$	=	Market clearing price with contracts, where $\hat{P}_s \geq 0$
$\hat{P}_s^c$	=	Customer price with contracts, where $\hat{P}_s^c \geq 0$
	=	$\omega\bar{P} + (1 - \omega)\hat{P}_s$
$\hat{\pi}_s^i$	=	Profit of genco i from the spot market with contracts signed, for $i = 1, 2, \ldots, n$

4.2.2 Assumptions

To simplify our model, we make two assumptions as follows. These two assumptions can be seen in Green (1999) and Rudkevich (2005).

Assumption 4-1. Demand function is linear with uncertainty.

We assume that the demand function is $Q(p) = a - bp$, where a and $b > 0$, are independent random variables.

Assumption 4-2. Production cost of each genco is a quadratic function of quantity produced.

We assume all gencos have the same cost and the production cost of genco i is $P_C(x_i) = Kcx_i^2 + c_1$, for $i = 1, 2, \ldots, n$, where $c > 0$. Without loss of generality, we let $K = 0.5$ to have marginal cost as $M_C(x_i) = cx_i$, for $i = 1, 2, \ldots, n$.

4.2.3 The SFE model

In the SFE model, each genco determines a supply function to maximize its own profit in the spot market. We assume that the linear supply function of genco i is $S_i(p) = \alpha_i + \beta_i p$, for $i = 1, 2, \ldots, n$, where α_i and β_i are given constants. Note that α_i and $\beta_i > 0$, are the intercept and slope of the supply function, respectively. For genco i that has a contract quantity, $\bar{Q}_i$, for $i = 1, 2, \ldots, n$, its profit from the spot market is

$$\hat{\pi}_s^i = p[(a - bp - S_{-i}(p)) - \bar{Q}_i] - P_C(a - bp - S_{-i}(p)),$$

where $p[(a - bp - S_{-i}(p)) - \bar{Q}_i]$ is the revenue from the spot market and $P_C(a - bp - S_{-i}(p))$ is the cost.

Consider that all gencos have linear supply functions. The equilibrium price in the spot market must match supply and demand. That is,

$$a - bp = \sum_{i=1}^{n} \alpha_i + \sum_{i=1}^{n} \beta_i p. \tag{4.1}$$

Define $\sum_{j \neq i} \beta_j = \sum_{j=1}^{n} \beta_j - \beta_i$. The equilibrium is described by the following first order condition:

$$\begin{aligned}
\frac{\partial \hat{\pi}_s^i}{\partial p} &= \frac{\partial \{p[(a-bp-S_{-i}(p))-\bar{Q}_i]-P_C(a-bp-S_{-i}(p))\}}{\partial p} \\
&= (a-bp-S_{-i}(p))-\bar{Q}_i+p\left(-b-\frac{\partial S_{-i}(p)}{\partial p}\right) \\
&\quad -c(a-bp-S_{-i}(p))\left(-b-\frac{\partial S_{-i}(p)}{\partial p}\right) \\
&= S_i(p)-\bar{Q}_i+(p-cS_i(p))\left(-b-\frac{\partial S_{-i}(p)}{\partial p}\right) \\
&= \alpha_i+\beta_i p-\bar{Q}_i+[p-c(\alpha_i+\beta_i p)](-b-\sum_{j\neq i}\beta_j) \qquad (4.2)\\
&= \alpha_i+\beta_i p-\bar{Q}_i+c(b+\sum_{j\neq i}\beta_j)\alpha_i+(1-c\beta_i)(-b-\sum_{j\neq i}\beta_j)p \\
&= -\bar{Q}_i+[1+c(b+\sum_{j\neq i}\beta_j)]\alpha_i+[\beta_i+(1-c\beta_i)(-b-\sum_{j\neq i}\beta_j)]p \\
&= -\bar{Q}_i+[1+c(b+\sum_{j\neq i}\beta_j)]\alpha_i+\{(-b-\sum_{j\neq i}\beta_j)+[1+c(b+\sum_{j\neq i}\beta_j)]\beta_i\}p \\
&= 0,
\end{aligned}$$

where Equation (4.2) holds from $S_i(p)=\alpha_i+\beta_i p$ and $P_C(x_i)=0.5cx_i^2+c_1$. For $i=1,2,\ldots,n$, we have

$$-\bar{Q}_i+[1+c(b+\sum_{j\neq i}\beta_j)]\alpha_i+\{(-b-\sum_{j\neq i}\beta_j)+[1+c(b+\sum_{j\neq i}\beta_j)]\beta_i\}p=0. \qquad (4.3)$$

Note that Equation (4.3) represents n equations. The above n equations can only hold together if and only if

$$\alpha_i = \frac{\bar{Q}_i}{1+c(b+\sum_{j\neq i}\beta_j)}, \qquad (4.4)$$

$$\text{and} \quad \beta_i = \frac{b+\sum_{j\neq i}\beta_j}{1+c(b+\sum_{j\neq i}\beta_j)}. \qquad (4.5)$$

From Equations (4.4) and (4.5), we can see that the values of α_i and β_i have no relationship with a.

From Equation (4.5), the bids of gencos are dependent on the value of b. For various possible value of b, the bids are different.

We now show that all gencos' supply functions have the same slope.

Lemma 4.1. $\beta_1 = \beta_2 = \cdots = \beta_n = \beta$.

Proof. From b and $\beta_i > 0$, we have $1 + c\beta_i(b + \sum_{j\neq i}\beta_j) > 0$ for $i = 1, 2, \ldots, n$. Then, from Equation (4.5), for genco h, we have

$$\beta_h + c\beta_h(b + \sum_{j\neq h}\beta_j) = b + \sum_{j\neq h}\beta_j. \tag{4.6}$$

Similarly, for genco k, we have

$$\beta_k + c\beta_k(b + \sum_{j\neq k}\beta_j) = b + \sum_{j\neq k}\beta_j. \tag{4.7}$$

From Equations (4.6) and (4.7), we have

$$\beta_h - \beta_k + c\beta_h(b + \sum_{j\neq h}\beta_j) - c\beta_k(b + \sum_{j\neq k}\beta_j) = \sum_{j\neq h}\beta_j - \sum_{j\neq k}\beta_j.$$

Then, we can simplify it as

$$\beta_h - \beta_k + bc\beta_h - bc\beta_k + c\beta_h \sum_{j\neq h,k}\beta_j - c\beta_k \sum_{j\neq k}\beta_j = \beta_k - \beta_h.$$

As a result,

$$(\beta_h - \beta_k)(2 + bc + c\sum_{j\neq h,k}\beta_j) = 0.$$

From b, c and $\beta_i > 0$, for $i = 1, 2, \ldots, n$, we have $\beta_h = \beta_k$ for any genco h and k. Hence, $\beta_1 = \beta_2 = \cdots = \beta_n = \beta$. □

Define $\lambda = 1/(b + n\beta)$, $\mu = 1/\{1 + c[b + (n-1)\beta]\}$. From Equation (4.1), the MCP is

$$\begin{aligned}
\hat{P}_s &= \frac{a - \sum_{i=1}^n \alpha_i}{b + \sum_{i=1}^n \beta_i} \\
&= \frac{a - \frac{\bar{Q}}{1+c[b+(n-1)\beta]}}{b + n\beta} &\quad (4.8)\\
&= \frac{a\{1 + c[b + (n-1)\beta]\} - \bar{Q}}{(b + n\beta)\{1 + c[b + (n-1)\beta]\}} &\quad (4.9)\\
&= \lambda a - \lambda\mu\bar{Q},
\end{aligned}$$

where Equation (4.8) holds from Equation (4.4) and Lemma 4.1. Consequently, the variance of MCP is

$$Var(\hat{P}_s) = E[(\hat{P}_s - E[\hat{P}_s])^2] = E[(\lambda a - \lambda\mu\bar{Q} - E[\lambda a - \lambda\mu\bar{Q}])^2].$$

The first partial derivative of $Var(\hat{P}_s)$ with respect to $\bar{Q}$ is

$$\begin{aligned}
\frac{\partial Var(\hat{P}_s)}{\partial \bar{Q}} &= \frac{\partial E[(\lambda a - \lambda\mu\bar{Q} - E[\lambda a - \lambda\mu\bar{Q}])^2]}{\partial \bar{Q}} \\
&= E[2(\lambda a - \lambda\mu\bar{Q} - E[\lambda a - \lambda\mu\bar{Q}]) \cdot (-\lambda\mu + E[\lambda\mu])] \\
&= 2E[(\lambda a - \lambda\mu\bar{Q} - E[\lambda a - \lambda\mu\bar{Q}]) \cdot (-\lambda\mu) \\
&\qquad +(\lambda a - \lambda\mu\bar{Q} - E[\lambda a - \lambda\mu\bar{Q}]) \cdot E[\lambda\mu]] \\
&= 2E[(\lambda a - \lambda\mu\bar{Q}) \cdot (-\lambda\mu) - E[\lambda a - \lambda\mu\bar{Q}] \cdot (-\lambda\mu) \\
&\qquad +(\lambda a - \lambda\mu\bar{Q}) \cdot E[\lambda\mu] - E[\lambda a - \lambda\mu\bar{Q}] \cdot E[\lambda\mu]] \\
&= 2E[(\lambda a - \lambda\mu\bar{Q}) \cdot (-\lambda\mu) + (\lambda a - \lambda\mu\bar{Q}) \cdot E[\lambda\mu]] \\
&= 2E[(\lambda a - \lambda\mu\bar{Q}) \cdot (-\lambda\mu + E[\lambda\mu])] \\
&= 2E[\hat{P}_s(-\lambda\mu + E[\lambda\mu])]. \qquad (4.10)
\end{aligned}$$

We now show the relationship between slope of supply function and slope of demand function.

Lemma 4.2. *β is an increasing function of b.*

Proof. From Equation (4.5) and Lemma 4.1, we have

$$\begin{aligned}
\beta &= \beta_i \\
&= \frac{b + \sum_{j\neq i}\beta_j}{1 + c(b + \sum_{j\neq i}\beta_j)} \\
&= \frac{b + (n-1)\beta}{1 + c[b + (n-1)\beta]}.
\end{aligned}$$

Then,

$$c(n-1)\beta^2 + (bc + 2 - n)\beta - b = 0. \qquad (4.11)$$

Solving the above equation, we get two solutions. Since we assume that $\beta > 0$, the negative solution of Equation (4.11) is infeasible. Hence, we have

$$\beta = -\frac{bc + 2 - n}{2c(n-1)} + \sqrt{\left(\frac{bc + 2 - n}{2c(n-1)}\right)^2 + \frac{b}{c(n-1)}}. \qquad (4.12)$$

Now, we show that $bc + 2 - n + n - 1 = bc + 1 > bc$. When both sides are divided by $c^2(n-1)$, we have

$$\frac{bc + 2 - n}{c^2(n-1)} + \frac{1}{c^2} > \frac{b}{c(n-1)}. \qquad (4.13)$$

Hence,

$$\begin{aligned}
\frac{bc+2-n}{2c(n-1)}+\frac{1}{c} &= \sqrt{\left(\frac{bc+2-n}{2c(n-1)}+\frac{1}{c}\right)^2} \\
&= \sqrt{\left(\frac{bc+2-n}{2c(n-1)}\right)^2+\frac{bc+2-n}{c^2(n-1)}+\frac{1}{c^2}} \\
&> \sqrt{\left(\frac{bc+2-n}{2c(n-1)}\right)^2+\frac{b}{c(n-1)}}, \qquad (4.14)
\end{aligned}$$

where Equation (4.14) holds from Equation (4.13). The first partial derivative of β with respect to b is

$$\begin{aligned}
\frac{\partial\beta}{\partial b} &= \frac{\partial}{\partial b}\left(-\frac{bc+2-n}{2c(n-1)}+\sqrt{\left(\frac{bc+2-n}{2c(n-1)}\right)^2+\frac{b}{c(n-1)}}\right) \\
&= -\frac{1}{2(n-1)}+\frac{1}{2(n-1)}\frac{\frac{bc+2-n}{2c(n-1)}+\frac{1}{c}}{\sqrt{\left(\frac{bc+2-n}{2c(n-1)}\right)^2+\frac{b}{c(n-1)}}} \qquad (4.15) \\
&\geq 0,
\end{aligned}$$

where the first equation holds from Equation (4.12) and the inequality holds from Equation (4.14). Hence, β is an increasing function of b. □

We now show the relationship between intercept of supply function and slope of demand function.

Lemma 4.3. *α_i is a decreasing function of b, for $i = 1, 2, \ldots, n$.*

Proof. The first partial derivative of α_i with respect to b is

$$\begin{aligned}
\frac{\partial\alpha_i}{\partial b} &= \frac{\partial}{\partial b}\left(\frac{\bar{Q}_i}{1+c(b+\sum_{j\neq i}\beta_j)}\right) \\
&= \frac{-\bar{Q}_i[c(1+\sum_{j\neq i}\frac{\partial\beta_j}{\partial b})]}{[1+c(b+\sum_{j\neq i}\beta_j)]^2} \\
&\leq 0,
\end{aligned}$$

where the first equation holds from Equation (4.4) and the inequality holds from

Lemma 4.2. □

Now, we show the monotonic property of λ and μ over the slope of demand function.

Lemma 4.4. *λ, μ are decreasing functions of b.*

Proof. We have $\lambda = 1/(b + n\beta)$ and $\mu = 1/\{1 + c[b + (n-1)\beta]\}$. From Lemma 4.2, we know that β is an increasing function of b. Hence, λ and μ are decreasing functions of b. □

We now show the relationship between MCP and slope of demand function.

Lemma 4.5. *For $\bar{Q} \to 0^+$, we have $\hat{P}_s$ is a decreasing function of b.*

Proof. For $\bar{Q} \to 0^+$, the first partial derivative of $\hat{P}_s$ with respect to b is

$$
\begin{aligned}
\frac{\partial \hat{P}_s}{\partial b} &= \frac{\partial}{\partial b}\left(\frac{a\{1 + c[b + (n-1)\beta]\} - \bar{Q}}{(b + n\beta)\{1 + c[b + (n-1)\beta]\}}\right) \\
&= \frac{\partial}{\partial b}\left(\frac{a - \bar{Q} + ac[b + (n-1)\beta]}{(b + n\beta)\{1 + c[b + (n-1)\beta]\}}\right) \\
&= \frac{1}{(b + n\beta)^2\{1 + c[b + (n-1)\beta]\}^2} \\
&\quad \cdot \left[ac\left(1 + (n-1)\frac{\partial\beta}{\partial b}\right)(b + n\beta)\{1 + c[b + (n-1)\beta]\}\right. \\
&\qquad -\{a - \bar{Q} + ac[b + (n-1)\beta]\}(b + n\beta)c\left(1 + (n-1)\frac{\partial\beta}{\partial b}\right) \\
&\qquad \left. -\{a - \bar{Q} + ac[b + (n-1)\beta]\}\left(1 + n\frac{\partial\beta}{\partial b}\right)\{1 + c[b + (n-1)\beta]\}\right] \\
&= \frac{1}{(b + n\beta)^2\{1 + c[b + (n-1)\beta]\}^2} \\
&\quad \cdot \left[\bar{Q}(b + n\beta)c\left(1 + (n-1)\frac{\partial\beta}{\partial b}\right)\right. \\
&\qquad \left. -\{a - \bar{Q} + ac[b + (n-1)\beta]\}\left(1 + n\frac{\partial\beta}{\partial b}\right)\{1 + c[b + (n-1)\beta]\}\right]
\end{aligned}
$$

$$\begin{aligned}
&= \frac{1}{(b+n\beta)^2\{1+c[b+(n-1)\beta]\}^2} \\
&\quad \cdot \left[\bar{Q}(b+n\beta)c\left(1+(n-1)\frac{\partial\beta}{\partial b}\right) - ac\left(1+n\frac{\partial\beta}{\partial b}\right)[b+(n-1)\beta] \right. \\
&\qquad -ac^2\left(1+n\frac{\partial\beta}{\partial b}\right)[b+(n-1)\beta]^2 \\
&\qquad \left. -(a-\bar{Q})\left(1+n\frac{\partial\beta}{\partial b}\right)\{1+c[b+(n-1)\beta]\} \right] \\
&= \frac{1}{(b+n\beta)^2\{1+c[b+(n-1)\beta]\}^2} \\
&\quad \cdot \left[-(a-\bar{Q})c\left(1+n\frac{\partial\beta}{\partial b}\right)[b+(n-1)\beta] + \bar{Q}c\left(\beta - b\frac{\partial\beta}{\partial b}\right) \right. \\
&\qquad -ac^2\left(1+n\frac{\partial\beta}{\partial b}\right)[b+(n-1)\beta]^2 \\
&\qquad \left. -(a-\bar{Q})\left(1+n\frac{\partial\beta}{\partial b}\right)\{1+c[b+(n-1)\beta]\} \right] \\
&< 0,
\end{aligned}$$

where the first equation holds from Equation (4.9) and the inequality holds from $b > 0$, $\beta > 0$ and Lemma 4.2. □

Next, we show that the variance of MCP is a decreasing function of contract quantity under two extreme conditions.

Theorem 4.1. *For $\bar{Q} \to 0^+$, $Var(\hat{P}_s)$ is a decreasing function of $\bar{Q}$.*

Proof. For $\bar{Q} \to 0^+$, we have

$$\begin{aligned}
\frac{\partial Var(\hat{P}_s)}{\partial \bar{Q}} &= 2E[\hat{P}_s(-\lambda\mu + E[\lambda\mu])] \\
&= 2E[\hat{P}_s E[\lambda\mu] - \hat{P}_s\lambda\mu] \\
&= 2E\left[\int \hat{P}_s dF(b) \int \lambda\mu dF(b) - \int \hat{P}_s\lambda\mu dF(b) \middle| a \right] \\
&\leq 0, \qquad\qquad (4.16)
\end{aligned}$$

where the first equation holds from Equation (4.10) and the second equation holds from Assumption 4-1 that a and b are random variables. Moreover, the Inequality (4.16) holds from Lemmas 4.4 and 4.5. From Lemma 4.4, λ and μ are decreasing functions of b. From Lemma 4.5, for $\bar{Q} \to 0^+$, $\hat{P}_s$ is a decreasing function of b.

Hence, by the Chebyshev integral inequality, Inequality (4.16) holds. As a result, for $\bar{Q} \to 0^+$, the variance of MCP is a decreasing function of contract quantity. □

Now, we show that the variance of CP is a decreasing function of contract quantity under two extreme conditions.

Theorem 4.2. *For $\bar{Q} \to 0^+$, $Var(\hat{P}_s^c)$ is a decreasing function of contract quantity, $\bar{Q}$.*

Proof. From Theorem 4.1, for $\bar{Q} \to 0^+$, we have the variance of MCP, $Var(\hat{P}_s)$ is a decreasing function of $\bar{Q}$. Since

$$\begin{aligned} Var(\hat{P}_s^c) &= Var\left(\omega\bar{P} + (1-\omega)\hat{P}_s\right) \\ &= Var(\hat{P}_s)(1-\omega)^2, \end{aligned}$$

for $\bar{Q} \to 0^+$, the variance of CP is a decreasing function of contract quantity. □

Theorem 4.1 shows that the variance of MCP is decreased with the introduction of bilateral contracts. Since the hedge price is constant, we know that the variance of CP is reduced with the introduction of bilateral contracts if hedge ratio is assumed to be constant.

4.2.4 The Cournot model

We consider a market where n gencos are in competition to supply electricity. They can only bid the quantities that they are willing to supply. For genco i with contract quantity, $\bar{Q}_i$, its profit from the spot market is

$$\hat{\pi}_s^i = \left(\frac{a-x}{b}\right)(x_i - \bar{Q}_i) - P_C(x_i),$$

where $\left(\frac{a-x}{b}\right)(x_i - \bar{Q}_i)$ is the revenue from spot market and $P_C(x_i)$ is the cost.

The equilibrium is described by the following first order condition:

$$\frac{\partial \hat{\pi}_s^i}{\partial x_i} = \left(\frac{a - x - (x_i - \bar{Q}_i)}{b}\right) - cx_i = 0, \quad \text{for} \quad i = 1, 2, \ldots, n,$$

where the above equation holds from $P_C(x_i) = 0.5cx_i^2 + c_1$.

Define $\sum_{j\neq i} \bar{Q}_j = \sum_{j=1}^{n} \bar{Q}_j - \bar{Q}_i$. We show the optimal production quantity for each individual genco.

Lemma 4.6. *For genco i with contract quantity $\bar{Q}_i$, the optimal production quantity is*

$$x_i^{**} = \frac{a + \left((n+bc)\bar{Q}_i - \sum_{j\neq i} \bar{Q}_j\right)/(1+bc)}{n+1+bc}. \tag{4.17}$$

Proof. The proof of this lemma is similar as Bushnell (2007). □

From Lemma 4.6 and $\bar{Q} = \sum_{i=1}^{n} \bar{Q}_i$, total market production quantity is

$$\hat{Q}_s = \sum_{i=1}^{n} x_i^{**} = \frac{na + \bar{Q}}{n+1+bc}, \tag{4.18}$$

where Equation (4.18) holds from Equation (4.17). The MCP is

$$\begin{aligned} \hat{P}_s &= \frac{a - \hat{Q}_s}{b} && (4.19)\\ &= \frac{a - \frac{na+\bar{Q}}{n+1+bc}}{b} && (4.20)\\ &= \frac{a(n+1+bc) - na - \bar{Q}}{b(n+1+bc)} \\ &= \frac{a + abc - \bar{Q}}{b(n+1+bc)} \\ &= \frac{a - \bar{Q}}{b(n+1+bc)} + \frac{abc}{b(n+1+bc)}, && (4.21) \end{aligned}$$

where Equation (4.19) holds from Assumption 4-1 and Equation (4.20) holds from Equation (4.18).

Define $\gamma = 1/[b(n+1+bc)]$. From Equation (4.21), we have

$$\hat{P}_s = \frac{a - \bar{Q}}{b(n+1+bc)} + \frac{abc}{b(n+1+bc)} = (a - \bar{Q})\gamma + abc\gamma.$$

Consequently, the variance of MCP is

$$Var(\hat{P}_s) = E[(\hat{P}_s - E[\hat{P}_s])^2] = E[((a-\bar{Q})\gamma + abc\gamma - E[(a-\bar{Q})\gamma + abc\gamma])^2].$$

The first partial derivative of $Var(\hat{P}_s)$ with respect to $\bar{Q}$ is

$$\begin{aligned}
\frac{\partial Var(\hat{P}_s)}{\partial \bar{Q}} &= E[2((a-\bar{Q})\gamma + abc\gamma - E[(a-\bar{Q})\gamma + abc\gamma])(-\gamma + E[\gamma])] \\
&= 2E[-(a-\bar{Q})\gamma^2 - abc\gamma^2 + E[(a-\bar{Q})\gamma]\gamma + E[abc\gamma]\gamma \\
&\qquad +(a-\bar{Q})\gamma E[\gamma] + abc\gamma E[\gamma] \\
&\qquad -E[(a-\bar{Q})\gamma]E[\gamma] - E[abc\gamma]E[\gamma]] \\
&= 2E[-(a-\bar{Q})\gamma^2] - 2E[abc\gamma^2] \\
&\qquad +2E[(a-\bar{Q})\gamma]E[\gamma] + 2E[abc\gamma]E[\gamma]] \\
&= 2E[(a-\bar{Q})\gamma(E[\gamma]-\gamma)] + 2E[abc\gamma(E[\gamma]-\gamma)]. \qquad (4.22)
\end{aligned}$$

We now show that the variance of MCP does not increase when bilateral contracts are signed.

Theorem 4.3. *$Var(\hat{P}_s)$ is a decreasing function of $\bar{Q}$.*

Proof. Under Assumption 4-1, we have

$$\begin{aligned}
E[(a-\bar{Q})\gamma(E[\gamma]-\gamma)] &= E[(a-\bar{Q})]E[\gamma(E[\gamma]-\gamma)] \\
&= E[(a-\bar{Q})]E[\gamma(E[\gamma]-\gamma) - E[\gamma](E[\gamma]-\gamma)] \\
&= E[(a-\bar{Q})]E[(\gamma - E[\gamma])(E[\gamma]-\gamma)] \\
&= -E[(a-\bar{Q})]E[(\gamma - E[\gamma])^2] \\
&= -E[(a-\bar{Q})]Var(\gamma) \\
&\leq 0, \qquad (4.23)
\end{aligned}$$

where the second equation holds from $E[E[\gamma]-\gamma)] = 0$.

Under Assumption 4-1, we have

$$\begin{aligned}
E[abc\gamma(E[\gamma]-\gamma)] &= E[ac]E[b\gamma(E[\gamma]-\gamma)] \\
&= E[ac](E[b\gamma]E[\gamma] - E[b\gamma^2]) \\
&= E[ac]\left(\int b\gamma dF(b) \int \gamma dF(b) - \int b\gamma^2 dF(b)\right) \\
&\leq 0, \qquad (4.24)
\end{aligned}$$

where the inequality holds from the Chebyshev integral inequality since both γ and $b\gamma$ are decreasing functions of b.

From Equation (4.22), we have

$$\begin{aligned}\frac{\partial Var(\hat{P}_s)}{\partial \bar{Q}} &= 2E[(a-\bar{Q})\gamma(E[\gamma]-\gamma)] + 2E[abc\gamma(E[\gamma]-\gamma)] \\ &\leq 0,\end{aligned}$$

where the inequality holds from Equations (4.23) and (4.24). Hence, the variance of MCP is a decreasing function of contract quantity. □

Now, we show that the variance of CP is a decreasing function of contract quantity.

Theorem 4.4. *$Var(\hat{P}_s^c)$ is a decreasing function of $\bar{Q}$.*

Proof. From Theorem 4.3, we have $Var(\hat{P}_s)$ is a decreasing function of $\bar{Q}$. Since

$$\begin{aligned}Var(\hat{P}_s^c) &= Var\left(\omega\bar{P} + (1-\omega)\hat{P}_s\right) \\ &= Var(\hat{P}_s)(1-\omega)^2,\end{aligned}$$

the variance of CP is a decreasing function of contract quantity. □

Theorem 4.3 shows that the variance of MCP is decreased with the introduction of bilateral contracts. Since the hedge price is constant, we know that the variance of CP is reduced with the introduction of bilateral contracts if hedge ratio is assumed to be constant.

4.3 Numerical study

In Chapter 3, demand is assumed to be inelastic which is independent of price. However, demand function is a linear function in Chapter 4. In the Singapore electricity market, demand quantity is forecasted as a constant value which is independent of price. It can be interpreted as a special case of a linear demand function. In Singapore, gencos make offers to sell electricity in the market. An offer specifies the price at which an amount of electricity that a genco is willing to sell (Energy Market Authority (EMA) Singapore, 2010a). The SFE is much closer to the Singapore market in this aspect.

Continuing from Section 3.4, we now investigate the Singapore electricity market to see if it follows the Assumption 4-2 which states that production cost is a quadratic function of quantity produced. Secondly, we verify the main results in this chapter, that is, the coefficient of variation of CP is a decreasing function of hedge ratio. In addition, we discuss the relationship between coefficient of variation of USEP and hedge price.

Table 4.2: Mean of USEP and mean of demand for different demand intervals

Year	Demand interval	Mean of USEP	Mean of demand
2003	$[2535, 3158]$	70	3034
	$(3158, 3305]$	77	3232
	$(3305, 3462]$	80	3385
	$(3462, 3608]$	84	3537
	$(3608, 3793]$	86	3697
	$(3793, 4082]$	89	3925
	$(4082, 4288]$	93	4194
	$(4288, 4529]$	97	4408
	$(4529, 4644]$	107	4591
	$(4644, 4999]$	117	4732
2004	$[2691, 3308]$	67	3194
	$(3308, 3473]$	74	3391
	$(3473, 3637]$	78	3555
	$(3637, 3784]$	83	3709
	$(3784, 3972]$	81	3874
	$(3972, 4245]$	81	4099
	$(4245, 4470]$	83	4365
	$(4470, 4682]$	82	4577
	$(4682, 4816]$	87	4752
	$(4816, 5091]$	92	4908
2005	$[2879, 3496]$	84	3345
	$(3496, 3640]$	96	3568
	$(3640, 3799]$	109	3715
	$(3799, 3953]$	110	3877
	$(3953, 4146]$	106	4043
	$(4146, 4400]$	108	4266
	$(4400, 4648]$	113	4537
	$(4648, 4868]$	107	4756
	$(4868, 5027]$	111	4953
	$(5027, 5359]$	135	5110

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Table 4.2: Mean of USEP and mean of demand for different demand intervals– Continued

Year	Demand interval	Mean of USEP	Mean of demand
2006	$[3021, 3600]$	105	3452
	$(3600, 3753]$	115	3679
	$(3753, 3920]$	124	3834
	$(3920, 4091]$	129	4006
	$(4091, 4279]$	130	4178
	$(4279, 4556]$	135	4406
	$(4556, 4815]$	147	4695
	$(4815, 5030]$	130	4921
	$(5030, 5203]$	137	5125
	$(5203, 5452]$	153	5280
2007	$[3208, 3754]$	104	3617
	$(3754, 3919]$	111	3838
	$(3919, 4108]$	116	4012
	$(4108, 4292]$	122	4203
	$(4292, 4487]$	124	4382
	$(4487, 4763]$	131	4614
	$(4763, 5024]$	126	4901
	$(5024, 5223]$	125	5125
	$(5223, 5375]$	130	5302
	$(5375, 5782]$	140	5478
2008	$[3082, 3753]$	118	3607
	$(3753, 3941]$	151	3848
	$(3941, 4130]$	156	4030
	$(4130, 4323]$	160	4230
	$(4323, 4531]$	163	4425
	$(4531, 4801]$	164	4658
	$(4801, 5084]$	168	4951
	$(5084, 5280]$	161	5183
	$(5280, 5466]$	176	5378
	$(5466, 5949]$	201	5572
2009	$[3053, 3758]$	82	3547
	$(3758, 3956]$	114	3865
	$(3956, 4139]$	123	4046
	$(4139, 4345]$	132	4246
	$(4345, 4556]$	140	4449
	$(4556, 4805]$	140	4680
	$(4805, 5093]$	139	4952
	$(5093, 5289]$	141	5191
	$(5289, 5528]$	180	5415
	$(5528, 5876]$	239	5638
2010	$[3577, 4154]$	132	4022
	$(4154, 4327]$	142	4241
	$(4327, 4541]$	155	4427
	$(4541, 4754]$	155	4652
	$(4754, 4961]$	160	4853
	$(4961, 5231]$	172	5086
	$(5231, 5506]$	176	5380
	$(5506, 5721]$	180	5613
	$(5721, 5899]$	196	5813
	$(5899, 6294]$	232	5999

4.3.1 Production cost

From Assumption 4-2, the production cost of each genco is a quadratic function of quantity produced. The relationship between USEP and demand usually can reflect the relationship between production cost and demand. To avoid irrational offers, we only select those periods with USEP within [0, S$1000/MWh] for analysis. The model used to verify that production cost is a quadratic function is as follows:

$$\text{USEP} = b \cdot (\text{Demand})^2.$$

We partition the selected data of each year into 10 groups of equal size according to percentiles of demand. The mean of USEP and mean of demand in different demand intervals are presented in Table 4.2. The regression result of the model shows that b is 6.17×10^{-6}. Its significance level is less than 0.01. Moreover, we have $R^2 = 0.96$.

To further exam the relationship between price and demand, the original data from 2003 to 2010 with USEP within [0, S$1000/MWh] is used. The regression result of the model by using original data shows that b is 6.17×10^{-6}. Its significance level is less than 0.01. Moreover, we have $R^2 = 0.84$. This is quite close to results by using the data grouped according to demand level. Hence, the electricity price is statistically a quadratic function of demand. This result supports that the production cost could be a quadratic function of demand.

4.3.2 Coefficient of variation of CP and hedge ratio

We now use the data of Singapore electricity market to verify Theorems 4.2 and 4.4 under the environment of linear demand function and quadratic production cost.

From Table 3.10, the mean for coefficients of variations of CP with no hedge is 0.28. Moreover, the means for coefficients of variations of CP with 55% and 65% hedge ratios are 0.15 and 0.10, respectively. Hence, the coefficient of variation of CP is a decreasing function of hedge ratio. This result supports Theorems 4.2 and

4.4.

To further verify Theorems 4.2 and 4.4, we conduct one test: Test 4.1. In Test 4.1, we verify the relationships between coefficients of variations of CP with 55% and 65% hedge ratios.

Test 4.1: Coefficient of variation of CP with 65% hedge ratio is less than coefficient of variation of CP with 55% hedge ratio

From 2004 to Quarter 2, 2007, the hedge ratio is 65%. The quarterly coefficients of variations of USEP from 2004 to Quarter 2, 2007 are presented in Table 3.9. From Quarter 3, 2007 to 2010, the hedge ratio is 55%. Also, the quarterly coefficients of variations of USEP from Quarter 3, 2007 to 2010 are presented in Table 3.9.

Now, we test if the coefficient of variation of CP with 65% hedge ratio is less than the coefficient of variation of CP with 55% hedge ratio. The null hypothesis H_0 is that the mean for coefficients of variations of CP with 65% hedge ratio is equal to the mean for coefficients of variations of CP with 55% hedge ratio. That is

$$\begin{aligned} H_0 : \bar{X}_H &= \bar{X}_L, \\ H_1 : \bar{X}_H &< \bar{X}_L, \end{aligned}$$

where $\bar{X}_L$ is the mean for coefficients of variations of CP with 55% hedge ratio and $\bar{X}_H$ is the mean for coefficients of variations of CP with 65% hedge ratio.

In this test, we set the significance level at 0.05. The probability of Type I error is 0.03, which is less than the significance level. Thus, we reject H_0. Statistically, the mean for coefficients of variations of CP with 65% hedge ratio is significantly less than the mean for coefficients of variations of CP with 55% hedge ratio. Moreover, we have the coefficients of variations of CP with 65% and 55% hedge ratios are significantly less than the coefficient of variation of CP with no hedge from Tests 3.1 and 3.2 in Chapter 3. Hence, we have coefficient of variation of CP is a decreasing function of hedge ratio. This result supports Theorems 4.2 and 4.4.

Table 4.3: Coefficient of variation of USEP for selected periods

Year	Quarter	Hedge price ($/MWh)	Hedge ratio	Coefficient of variation of USEP
2003	Q1	94.24	0	0.28
	Q2	96.25	0	0.21
	Q3	95.73	0	0.24
	Q4	101.56	0	0.39
2004	Q1	94.24	0.65	0.11
	Q2	96.25	0.65	0.21
	Q3	95.73	0.65	0.14
	Q4	101.56	0.65	0.46
2005	Q1	101.29	0.65	0.40
	Q2	96.35	0.65	0.40
	Q3	117.38	0.65	0.48
	Q4	128.39	0.65	0.39
2006	Q1	140.7	0.65	0.27
	Q2	139.44	0.65	0.43
	Q3	147.9	0.65	0.48
	Q4	150.2	0.65	0.42
2007	Q1	134.66	0.65	0.54
	Q2	121.14	0.65	0.23
	Q3	137.25	0.55	0.25
	Q4	150.04	0.55	0.10
2008	Q1	161.8	0.55	0.29
	Q2	174.44	0.55	0.28
	Q3	183.25	0.55	0.16
	Q4	238.64	0.55	0.42
2009	Q1	167.14	0.55	0.45
	Q2	115.26	0.55	0.64
	Q3	138.92	0.55	0.44
	Q4	161.7	0.55	0.42
2010	Q1	171.05	0.55	0.49
	Q2	176.1	0.55	0.29
	Q3	176.29	0.55	0.20
	Q4	165.71	0.55	0.35

4.3.3 Coefficient of variation of USEP and hedge ratio

There is no hedge in 2003. From 2004 to Quarter 2, 2007, the hedge ratio is 65%. From Quarter 3, 2007 to 2010, the hedge ratio is 55%. The quarterly coefficients of variations of USEP from 2003 to 2010 are presented in Table 4.3. To examine the variation of USEP with and without hedge, we conduct three tests: Test 4.2, Test 4.3 and Test 4.4. In Test 4.2, we verify the relationship between coefficients of variations of USEP with no hedge ratio and with 65% hedge ratio. In Test 4.3, we verify the relationship between coefficients of variations of USEP with no hedge ratio and with 55% hedge ratio. In Test 4.4, we verify the relationships between coefficients of variations of USEP with 55% and 65% hedge ratios.

Test 4.2: Relationship between coefficients of variations of USEP with no hedge ratio and with 65% hedge ratio

The null hypothesis H_0 is that the mean for coefficients of variations of USEP with 65% hedge ratio is equal to the mean for coefficients of variations of USEP with no hedge. That is

$$\begin{aligned} H_0 : \bar{X}_H^U &= \bar{X}^U, \\ H_1 : \bar{X}_H^U &\neq \bar{X}^U, \end{aligned}$$

where $\bar{X}^U$ is the mean for coefficients of variations of USEP with no hedge and $\bar{X}_H^U$ is the mean for coefficients of variations of USEP with 65% hedge ratio.

In this test, we set the significance level at 0.32. The probability of Type I error is 0.02, which is greater than the significance level. Thus, we can not reject H_0. Consequently, mean for coefficients of variations of USEP with 65% hedge ratio does not have a statistically significant relationship with mean for coefficients of variations of USEP with no hedge.

Test 4.3: Relationship between coefficients of variations of USEP with no hedge ratio and with 55% hedge ratio

The null hypothesis H_0 is that the mean for coefficients of variations of USEP with 55% hedge ratio is equal to the mean for coefficients of variations of USEP with no hedge. That is

$$\begin{aligned} H_0 : \bar{X}_L^U &= \bar{X}^U, \\ H_1 : \bar{X}_L^U &\neq \bar{X}^U, \end{aligned}$$

where $\bar{X}^U$ is the mean for coefficients of variations of USEP with no hedge and $\bar{X}_L^U$

is the mean for coefficients of variations of USEP with 55% hedge ratio.

In this test, we set the significance level at 0.22. The probability of Type I error is 0.01, which is less than the significance level. Thus, we can not reject H_0. Consequently, mean for coefficients of variations of USEP with 55% hedge ratio does not have a statistically significant relationship with mean for coefficients of variations of USEP with no hedge.

Test 4.4: Relationships between coefficients of variations of USEP with 55% and 65% hedge ratios

The null hypothesis H_0 is that the mean for coefficients of variations of USEP with 65% hedge ratio is equal to the mean for coefficients of variations of USEP with 55% hedge ratio. That is

$$\begin{aligned} H_0 : \bar{X}_H^U &= \bar{X}_L^U, \\ H_1 : \bar{X}_H^U &\neq \bar{X}_L^U, \end{aligned}$$

where $\bar{X}_L^U$ is the mean for coefficients of variations of USEP with 55% hedge ratio and $\bar{X}_H^U$ is the mean for coefficients of variations of USEP with 65% hedge ratio.

In this test, we set the significance level at 0.83. The probability of Type I error is 0.03, which is less than the significance level. Thus, we can not reject H_0. Consequently, mean for coefficients of variations of USEP with 65% hedge ratio does not have a statistically significant relationship with mean for coefficients of variations of USEP with 55% hedge ratio. From Test 4.2, Test 4.3 and Test 4.4, we observe that whether the variation of MCP decreases or increases as hedge quantity increases is inconclusive.

4.4 Conclusions

In this chapter, we consider an environment where demand is uncertain. We formulate the spot market as SFE and Cournot models and examine the impact of bilateral contracts on the uncertainties of MCP and CP. The results show that the variances of MCP and CP are decreasing functions of contract quantity in a competitive market. Even when the market is not competitive, bilateral contracts can also reduce the variances of MCP and CP if the hedge quantity is within a reasonable range. Also, a numerical study is conducted using data from the Singapore electricity market from 2003 to 2010 to verify our models.

Chapter 5

Market Power in the Electricity Market

5.1 Introduction

The electricity industries have been deregulated since the 1980s in many countries, such as United Kingdom, Singapore, New Zealand and Spain (Joskow, 2008). One important part in the deregulation is the introduction of electricity spot market. The electricity spot market is a real time market where electricity is traded. There are three participants in the electricity spot market: suppliers, buyers and ISO. Generally, the suppliers are gencos. The buyers include retailers and real consumers. Retailers buy electricity from the spot market and sell it to real consumers who do not have economical scale to buy electricity directly from the spot market. The ISO dispatches electricity for each period in electricity spot markets, such as in United Kingdom and Singapore. Usually, gencos offer their electricity at different prices. Then, the ISO collects these offers and the demand from the buyer side. After that, the ISO dispatches cheap electricity first to meet the demand.

The deregulated electricity markets are expected to be stable and competitive. However, MCP may be unstable in the deregulated electricity markets due to the three major contributors. Firstly, the offer behavior of gencos may cause the volatility of MCP. There are some factors affecting the offer behavior of gencos, such as the production cost and the bidding strategies of gencos. Among these factors, the most volatile factor is the production cost. Secondly, the demand behavior of buyers may cause the volatility of MCP. The inelasticity and fluctuation of demand are the two attributes of electricity buyers (Stoft, 2002). Finally, lack of inventory for electricity also contributes to the volatility of MCP. Since electricity may be too expensive to store, gencos may raise the price as high as they like when demand is unexpectedly high.

Apart from the volatility of MCP, another issue of the deregulated electricity markets is the competitiveness of the markets. The suppliers may have market power which reduces the market competition. In a perfect competitive market, each genco bids according to its marginal cost (MC). However, in a monopoly market or an oligopoly market, gencos adopt bidding strategies to maximize their profits. This ability of a genco to use bidding strategies is called market power (Wolak, 2000). The genco with market power is usually called the price maker (De La Torre et al., 2002). Generally, price makers can have influence on the electricity prices and earn more profit.

The deregulated electricity markets are expected to be competitive. There are two reasons. The first reason is that the competition encourages gencos to control operating costs and improve technologies in the spot markets. Secondly, the benefit of competition from the deregulated electricity markets can be shared by consumers (Joskow, 2008). However, market power does exist in some electricity markets. For example, Woo et al. (2003) examined the market power in the United Kingdom, Norway, Alberta and California electricity markets. They found that market power exists in all these four markets. Mount (2001) showed that the gencos with market power can increase electricity prices. In this case, consumers suffer from high electricity prices.

The measurement of market power attracts much attention recently. Researchers propose many indexes to measure market power. There are two types of indexes: structural indexes and behavioral indexes. The structural indexes show the measurement of market concentration level. Usually, we use the market share proportions for gencos to generate the structural indexes. The structural indexes include the CR, HHI, SMA, RSI, MRR and DM (Gan and Bourcier, 2002; Chang, 2007; Melnik et al., 2008). The behavioral indexes are measurements of market power which directly use the data of prices and profits. They include Lerner Index and the variations of Lerner Index (Stoft, 2002; Nanduri and Das, 2007). Recently, the data of profits is directly used to measure market power. For example, Nanduri and Das (2007) proposed the RMPI, which uses the data of profits.

Many researchers use structural and behavioral indexes to measure market power. For example, MRR is used to test the market power in the United Kingdom electricity market (Gan and Bourcier, 2002). Chang (2007) used CR, HHI, SMA, RSI and Lerner Index to measure market power in the Singapore electricity market. Ciarreta and Esponosa (2009) used the lower bound of Lerner Index to test if market power exists in the Spain electricity market. Hellmer and Wårell (2009) used HHI and DM to measure market power in the Nordic electricity market.

The structural indexes use only the data of market shares, while the behavioral indexes use only the data of the market prices and MCs. Although RMPI directly considers the data of profits as the measurement of market power, we are interested in finding the relative increase in profit to measure market power. Since market power is the ability of a genco to maximize its own profit, we should investigate profit directly. Therefore, we propose an index using data of profits to measure market power in this chapter.

In order to control the volatility of MCP and to mitigate market power, vesting contracts and forward contracts are introduced. These two types of contracts work similarly in the spot markets. Contract price and contract quantity are the two basic elements of these contracts. The difference is that the forward contracts are negotiated while the vesting contracts are not. In this chapter, the bilateral contracts can be either vesting contracts or forward contracts. This is because, before bidding in the spot markets, gencos may sign one or both types of these contracts.

No matter what type of contract is signed, it represents an agreement on dispatching an amount of electricity (contract quantity) at a fixed price (contract price). Bilateral contract is one method used to control volatility of MCP. Chang and Park (2007) showed that bilateral contracts may reduce the variance of electricity prices in Singapore. Moreover, bilateral contracts can also control market power. Kelman et al. (2001) showed that the introduction of bilateral contracts can reduce market power, which is measured by the data of market prices.

In this chapter, we consider Cournot models and study the impact of bilateral contracts on the spot markets. It is well known that Cournot models are studied by many researchers. Allaz and Vila (1993) presented Cournot models with two suppliers and investigated the impact of bilateral contracts on the market prices. Bushnell (2007) extended this model and considered a situation with multiple gencos. Recently, some researchers provide a modern model for both markets: the spot market and the forward market. Since forward contracts are signed before the spot market, these two markets are usually modeled as a two stage equilibrium problem with equilibrium constraints. When signing the forward contracts, each genco must consider equilibrium constraints from the spot markets (Yao et al., 2008). Su (2007) showed the existence of solutions for the deterministic two stage equilibrium problem with equilibrium constraints. Zhang et al. (2010) proposed a stochastic equilibrium problem with equilibrium constraints where market demand is uncertain. They discussed the relationship between spot and forward markets.

Now, we introduce the market rules. Firstly, the gencos may sign bilateral contracts with contract price and contract quantity before the spot market. These contracts are financial bilateral contracts. A financial bilateral contract considers the difference between contract and MCP. If MCP is higher than contract price, consumers get credit from gencos. Otherwise, customers pay the debit to gencos. As a result, contract quantity is equivalently sold at contract price. Moreover, any MCQ other than contract quantity is called SMQ, which is sold at MCP. Besides, we are also concerned with the price combined by contract price and MCP which is called CP. It is a trading-quantity weighted price. More details about the market rules can be found in Niu et al. (2005) and Bushnell (2007).

In this chapter, we study Cournot models to investigate the impact of bilateral contracts on the spot market. The MCQ, SMQ, MCP, CP, profit of the market and market power in the spot market are examined closely. The remaining parts of this chapter are organized as follows. In Section 5.2, we adopt Cournot models to formulate the spot market with bilateral contracts. We study the MCQ, SMQ, MCP, CP and profit of the spot market. In Section 5.2, we also study market power measured by both Lerner Index and Profit Index. Results from these two indexes are also showed in Section 5.2. In Section 5.3, a numerical study is conducted to verify our models. Conclusions and directions for further research are presented in Section 5.4.

5.2 The models

In this section, we consider a market with multiple gencos. Gencos can sign bilateral contracts before the spot market. In the spot market, each genco always tries to maximize its own profit. We examine the impact of bilateral contracts on the spot market with Cournot models. We assume that the market demand and the production cost are linear functions. The assumption of linear demand function can be found in many works, for example, Bushnell (2007) and Yao et al. (2008). Moreover, the assumption of production cost can be found in Allaz and Vila (1993) and Sapio and Wylomanska (2008). Notations used in the spot market without and with contracts are showed in Tables 5.1 and 5.2, respectively.

Table 5.1: Notations used in the spot market without contracts

n	$=$	Number of generation companies (gencos)
x_i	$=$	Electricity quantity produced by genco i, for $i = 1, 2, \ldots, n$, where $x_i \geq 0$
x	$=$	Total electricity quantity produced, where $x = \sum_{i=1}^{n} x_i$
$P(x)$	$=$	Price that consumers are willing to pay at quantity x without contracts
Q_s	$=$	Market clearing quantity without contracts, where $Q_s \geq 0$
P_s	$=$	Market clearing price without contracts, where $P_s \geq 0$
P_s^c	$=$	Customer price without contracts, where $P_s^c \geq 0$
	$=$	P_s
$\pi_n^i(x_i)$	$=$	Net profit of genco i when producing electricity quantity x_i without contracts, for $i = 1, 2, \ldots, n$, where $\pi_n^i(x_i) > 0$

Table 5.2: Notations used in the spot market with contracts

$\bar{P}$	$=$	Contract price, where $\bar{P} \geq 0$
$\bar{Q}_i$	$=$	Contract quantity for genco i, for $i = 1, 2, \ldots, n$, where $\bar{Q}_i > 0$
$\bar{Q}$	$=$	Total contract quantity, where $\bar{Q} = \sum_{i=1}^{n} \bar{Q}_i > 0$
y	$=$	Bidding quantity in the spot market with contract quantity $\bar{Q}$, where $y \geq 0$
	$=$	$x - \bar{Q}$
$\hat{P}(y)$	$=$	Price that consumers are willing to pay at bidding quantity y with contracts
$\hat{Q}_s$	$=$	Market clearing quantity with contracts, where $\hat{Q}_s \geq \bar{Q} > 0$
ω	$=$	Hedge ratio, where $\omega \in (0, 1]$
	$=$	$\bar{Q}/\hat{Q}_s$
$\hat{P}_s$	$=$	Market clearing price with contracts, where $\hat{P}_s \geq 0$
$\hat{P}_s^c$	$=$	Customer price with contracts, where $\hat{P}_s^c \geq 0$
	$=$	$\omega\bar{P} + (1-\omega)\hat{P}_s$
$\hat{\pi}_n^i(x_i)$	$=$	Net profit of genco i when producing electricity quantity x_i with contracts, for $i = 1, 2, \ldots, n$, where $\hat{\pi}_n^i(x_i) > 0$

5.2.1 Assumptions

We make the following two assumptions in our model.

Assumption 5-1. Demand function is linear with uncertainty.

Generally, the marginal utility of electricity is decreasing on quantity purchased (McConnell and Brue, 2008). Thus, price can be considered as a decreasing function of quantity. We assume that the demand function without contracts is

$$P(x) = a - bx + \epsilon, \tag{5.1}$$

where a and $b > 0$ and ϵ is a random variable with $Var(\epsilon) = \sigma^2$. Note that $a+\epsilon > 0$. Otherwise, the price is not larger than 0. In this case, not electricity will be traded. We assume uncertainty is only in the intercept of the demand function. However, we assume that uncertainty is in both the intercept and slope of the demand in Chapter 4.

With contracts, an amount of electricity equal to contract quantity, $\bar{Q}$, is satisfied by contracts. Without loss of generality, we have the demand function with contracts as

$$\begin{aligned} \hat{P}(y) &= P(y + \bar{Q}) \\ &= a - b(y + \bar{Q}) + \epsilon \\ &= a - by + \epsilon - b\bar{Q}. \end{aligned} \tag{5.2}$$

From the definition of $\hat{P}(y)$, we have

$$\hat{P}(x - \bar{Q}) = P(x). \tag{5.3}$$

From Equation (5.3), we know that the model where demand function is changed is the same as the model where demand function is unchanged. Therefore, the assumption that demand function is unchanged with the introduction of bilateral contracts is justified.

Assumption 5-2. Production cost of each genco is a linear function of quantity produced.

We assume that the production cost of genco i is $c_i x_i$, where c_i is the marginal cost of genco i, for $i = 1, 2, \ldots, n$. Without loss of generality, we assume $0 \leq c_i \leq c_{i+1}$, for $i = 1, 2, \ldots, n-1$. From microeconomics, we have the trading quantity only if $c_i \leq P(x)$. If $c_i > a + \epsilon - b$, then

$$a + \epsilon - b < c_i \leq P(x) = a + \epsilon - bx.$$

From the above inequality, we have $x < 1$. This implies no meaningful trading quantity. As a result, we always assume $c_i \leq a + \epsilon - b$. Moreover, we assume that $\lim_{n\to\infty} c_n = \bar{c}$.

5.2.2 Electricity market without bilateral contracts

We consider an electricity market with n gencos and no bilateral contracts are allowed in this section. Without contracts, the profit of genco i, is

$$\begin{aligned} \pi_n^i(x_i) &= P(x) \cdot x_i - c_i x_i \\ &= -b{x_i}^2 + \left\{ a + \epsilon - c_i - b \left[\left(\sum_{k=1}^{n} x_k \right) - x_i \right] \right\} x_i, \end{aligned} \tag{5.4}$$

where $P(x) \cdot x_i$ is the revenue and $c_i x_i$ is the production cost. Equation (5.4) holds from Equation (5.1).

Without contracts, the profit of genco i, is a quadratic function of quantity. Hence, the genco can maximize its profit by solving the quadratic function. Define the optimal solution of Equation (5.4) as x_i^*. We have the optimal quantity of genco i as

$$x_i^* = \frac{a + \epsilon - c_i - b\left[\left(\sum_{k=1}^{n} x_k^*\right) - x_i^*\right]}{2b}, \quad \text{for} \quad i = 1, 2, \ldots, n. \tag{5.5}$$

By summing up optimal quantities of all gencos, we have

$$\begin{aligned} \sum_{i=1}^{n} x_i^* &= \sum_{i=1}^{n} \frac{a + \epsilon - c_i - b\left[\left(\sum_{k=1}^{n} x_k^*\right) - x_i^*\right]}{2b} \\ &= \frac{n(a+\epsilon) - \left(\sum_{i=1}^{n} c_i\right) - (n-1)b\left(\sum_{k=1}^{n} x_k^*\right)}{2b}. \end{aligned}$$

Hence, the MCQ without contracts is

$$Q_s = \sum_{i=1}^{n} x_i^* = \frac{n(a+\epsilon) - \sum_{k=1}^{n} c_k}{(n+1)b}. \tag{5.6}$$

Note that Q_s is also the SMQ as contract quantity is 0. From Equations (5.5) and (5.6), we have

$$\begin{aligned} x_i^* &= \frac{a+\epsilon-c_i-b\left[\left(\sum_{k=1}^{n} x_k^*\right)-x_i^*\right]}{2b} \\ &= \frac{a+\epsilon-(n+1)c_i+\sum_{k=1}^{n} c_k}{(n+1)b}, \quad \text{for} \quad i=1,2,\ldots,n. \end{aligned} \tag{5.7}$$

As a result, the MCP without contracts is

$$\begin{aligned} P_s &= P(Q_s) && (5.8) \\ &= a - b\cdot\left(\frac{n(a+\epsilon)-\sum_{k=1}^{n} c_k}{(n+1)b}\right)+\epsilon && (5.9) \\ &= \frac{a+\epsilon+\sum_{k=1}^{n} c_k}{n+1}, && (5.10) \end{aligned}$$

where Equation (5.9) holds from Equations (5.1) and (5.6), and its variance is

$$Var(P_s) = Var\left(\frac{a+\epsilon+\sum_{k=1}^{n} c_k}{n+1}\right) = \frac{\sigma^2}{(n+1)^2}. \tag{5.11}$$

Moreover, the net profit of genco i without contracts is

$$\begin{aligned} \pi_n^i(x_i^*) &= P(Q_s)\cdot x_i^* - c_i x_i^* \\ &= (P_s - c_i)x_i^* && (5.12) \\ &= \left(\frac{a+\epsilon+\sum_{k=1}^{n} c_k}{n+1} - c_i\right)\left(\frac{a+\epsilon-(n+1)c_i+\sum_{k=1}^{n} c_k}{(n+1)b}\right) && (5.13) \\ &= \frac{\left[a+\epsilon-(n+1)c_i+\sum_{k=1}^{n} c_k\right]^2}{(n+1)^2 b}, \end{aligned}$$

where Equation (5.12) holds from Equation (5.8) and Equation (5.13) holds from Equations (5.7) and (5.10). Thus, the total profit of the market is

$$\sum_{i=1}^{n} \pi_n^i(x_i^*) = \sum_{i=1}^{n} \frac{\left[a+\epsilon-(n+1)c_i+\sum_{k=1}^{n} c_k\right]^2}{(n+1)^2 b}. \tag{5.14}$$

5.2.3 Electricity market with bilateral contracts

We now consider an electricity market with bilateral contracts and n gencos in this section. The profit of genco i with contracts is

$$\begin{aligned}\hat{\pi}_n^i(x_i) &= \hat{P}(x-\bar{Q})\cdot(x_i-\bar{Q}_i)+\bar{P}\bar{Q}_i-c_ix_i \\ &= -b{x_i}^2+\left\{a+\epsilon-c_i-b\left[\left(\sum_{k=1}^n x_k\right)-x_i\right]+b\bar{Q}_i\right\}x_i \\ &\quad +b\bar{Q}_i\left[\left(\sum_{k=1}^n x_k\right)-x_i\right]-(a+\epsilon)\bar{Q}_i+\bar{P}\bar{Q}_i, \end{aligned} \tag{5.15}$$

where $\hat{P}(x-\bar{Q})\cdot(x_i-\bar{Q}_i)$ is the revenue from selling electricity in the spot market and $\bar{P}\bar{Q}_i$ is the revenue from selling electricity through bilateral contracts. Moreover, c_ix_i is the production cost and Equation (5.15) holds from Equation (5.2).

With contracts, the profit of genco i, is a quadratic function of quantity. Hence, the genco can maximize its profit by solving the quadratic function. Define the optimal solution of Equation (5.15) as x_i^{**}. We have the optimal quantity of genco i as

$$x_i^{**}=\frac{a+\epsilon-c_i-b\left[\left(\sum_{k=1}^n x_k^{**}\right)-x_i^{**}\right]+b\bar{Q}_i}{2b}. \tag{5.16}$$

Similar to Equation (5.6), the MCQ with contracts is

$$\hat{Q}_s=\sum_{i=1}^n x_i^{**}=\frac{n(a+\epsilon)-\left(\sum_{k=1}^n c_k\right)+b\bar{Q}}{(n+1)b}. \tag{5.17}$$

Thus, the SMQ is

$$\begin{aligned}\hat{Q}_s-\bar{Q} &= \frac{n(a+\epsilon)-\sum_{k=1}^n c_k}{(n+1)b}-\frac{n\bar{Q}}{n+1} \\ &= Q_s-\frac{n\bar{Q}}{n+1} \\ &\geq 0, \end{aligned} \tag{5.18}$$

where the first equation holds from Equation (5.17) and Inequality (5.18) holds from $\hat{Q}_s\geq\bar{Q}$. Inequality (5.18) implies that

$$0<\bar{Q}\leq\frac{(n+1)Q_s}{n}=\frac{n(a+\epsilon)-\sum_{k=1}^n c_k}{nb}, \tag{5.19}$$

where the equation holds from Equation (5.6).

Now, we present the optimal quantity satisfied by genco i. Similar to Equation (5.7), we have

$$x_i^{**} = \frac{a + \epsilon - (n+1)c_i + (\sum_{k=1}^n c_k) + (n+1)b\bar{Q}_i - b\bar{Q}}{(n+1)b}. \tag{5.20}$$

As a result, the MCP with contracts is

$$\begin{aligned}
\hat{P}_s &= \hat{P}(\hat{Q}_s - \bar{Q}) &(5.21)\\
&= a - b \cdot \left(\frac{n(a+\epsilon) - (\sum_{k=1}^n c_k) + b\bar{Q}}{(n+1)b} \right) + \epsilon &(5.22)\\
&= \frac{a + \epsilon + (\sum_{k=1}^n c_k) - b\bar{Q}}{n+1}, &(5.23)
\end{aligned}$$

where Equation (5.22) holds from Equations (5.2) and (5.17). As a result, its variance is

$$Var(\hat{P}_s) = Var\left(\frac{a + \epsilon + (\sum_{k=1}^n c_k) - b\bar{Q}}{n+1} \right) = \frac{\sigma^2}{(n+1)^2}. \tag{5.24}$$

Moreover, the maximal net profit of genco i with contracts is

$$\begin{aligned}
\hat{\pi}_n^i(x_i^{**}) &= \hat{P}\left(\hat{Q}_s - \bar{Q}\right) \cdot (x_i^{**} - \bar{Q}_i) + \bar{P}\bar{Q}_i - c_i x_i^{**} \\
&= (\hat{P}_s - c_i)(x_i^{**} - \bar{Q}_i) + (\bar{P} - c_i)\bar{Q}_i &(5.25)\\
&= \left(\frac{a + \epsilon + (\sum_{k=1}^n c_k) - b\bar{Q}}{n+1} - c_i \right) \\
&\quad \cdot \left(\frac{a + \epsilon - (n+1)c_i + (\sum_{k=1}^n c_k) + (n+1)b\bar{Q}_i - b\bar{Q}}{(n+1)b} - \bar{Q}_i \right) \\
&\quad + (\bar{P} - c_i)\bar{Q}_i &(5.26)\\
&= \frac{[a + \epsilon - (n+1)c_i + (\sum_{k=1}^n c_k) - b\bar{Q}]^2}{(n+1)^2 b} + \left(\bar{P} - c_i\right)\bar{Q}_i, &(5.27)
\end{aligned}$$

where Equation (5.25) holds from Equation (5.21) and Equation (5.26) holds from Equations (5.20) and (5.23). Thus, the total profit of the market is

$$\sum_{i=1}^n \hat{\pi}_n^i(x_i^{**}) = \sum_{i=1}^n \frac{[a + \epsilon - (n+1)c_i + (\sum_{k=1}^n c_k) - b\bar{Q}]^2}{(n+1)^2 b} + \sum_{i=1}^n \left(\bar{P} - c_i\right)\bar{Q}_i. \tag{5.28}$$

We now show the impact of bilateral contracts on the MCQ and MCP.

Lemma 5.1.
$$
\begin{aligned}
(i) \quad \hat{Q}_s &= Q_s + \frac{\bar{Q}}{(n+1)}, \\
(ii) \quad \hat{P}_s &= P_s - \frac{b\bar{Q}}{(n+1)}.
\end{aligned}
$$

Proof. (i) From Equations (5.6) and (5.17),

$$
\begin{aligned}
\hat{Q}_s &= \frac{n(a+\epsilon) - (\sum_{k=1}^{n} c_k) + b\bar{Q}}{(n+1)b} \\
&= \frac{n(a+\epsilon) - \sum_{k=1}^{n} c_k}{(n+1)b} + \frac{\bar{Q}}{(n+1)} \\
&= Q_s + \frac{\bar{Q}}{(n+1)}.
\end{aligned}
$$

(ii) From Equations (5.10) and (5.23),

$$
\begin{aligned}
\hat{P}_s &= \frac{a+\epsilon+(\sum_{k=1}^{n} c_k) - b\bar{Q}}{n+1} \\
&= \frac{a+\epsilon+\sum_{k=1}^{n} c_k}{n+1} - \frac{b\bar{Q}}{(n+1)} \\
&= P_s - \frac{b\bar{Q}}{(n+1)}. \qquad \square
\end{aligned}
$$

From Lemma 5.1, we know that with contracts, MCQ is increased by $\bar{Q}/(n+1)$ and MCP is decreased by $b\bar{Q}/(n+1)$. The impact of bilateral contracts on the MCQ and MCP depends on the number of gencos. We leave the discussion about the impact of the number of gencos on the MCQ and MCP to Theorem 5.3.

We now present some properties about MCQ, SMQ and MCP. Firstly, we show that MCQ is an increasing function of contract quantity. Secondly, we show that MCP and SMQ are decreasing functions of contract quantity. Thirdly, we show that MCQ with contracts is the upper bound of MCQ without contracts and MCQ without contracts is the upper bound of SMQ. Fourthly, we show that MCP without contracts is greater than MCP with contracts. Finally, we show that the variances of MCP with and without bilateral contracts are identical.

Theorem 5.1.

(i) $\hat{Q}_s(\bar{Q})$ is an increasing function of $\bar{Q}$.

(ii) $\hat{P}_s(\bar{Q})$ *and* $\hat{Q}_s(\bar{Q}) - \bar{Q}$ *are decreasing functions of* $\bar{Q}$*.*

(iii) $\hat{Q}_s - \bar{Q} < Q_s < \hat{Q}_s$*.*

(iv) $P_s > \hat{P}_s$*.*

(v) $Var(P_s) = Var(\hat{P}_s)$*.*

Proof. We have (i), (ii) and (iii) holding directly from Lemma 5.1.

(iv) From Equations (5.10) and (5.23), we have

$$\begin{aligned} P_s &= \frac{a + \epsilon + \sum_{k=1}^{n} c_k}{n+1} \\ &> \frac{a + \epsilon + (\sum_{k=1}^{n} c_k) - b\bar{Q}}{n+1} \\ &= \hat{P}_s. \end{aligned}$$

(v) From Equations (5.11) and (5.24), we have

$$Var(P_s) = \frac{\sigma^2}{(n+1)^2} = Var(\hat{P}_s). \qquad \square$$

Theorem 5.1 shows that the variance of MCP is unchanged with the introduction of bilateral contracts. Next, we show that the variance of CP will be reduced with bilateral contracts.

Theorem 5.2.

$$Var(\hat{P}_s^c) < Var(P_s^c).$$

Proof. We have

$$\begin{aligned} Var(\hat{P}_s^c) &= Var\left(\omega \bar{P} + (1-\omega)\hat{P}_s\right) \\ &= Var(\hat{P}_s)(1-\omega)^2 && (5.29) \\ &< Var(\hat{P}_s) && (5.30) \\ &= Var(P_s) && (5.31) \\ &= Var(P_s^c), && (5.32) \end{aligned}$$

where Equation (5.29) holds because ω and $\bar{P}$ are constants. Inequality (5.30) holds from $\omega \in (0, 1]$. Moreover, Equation (5.31) holds from Theorem 5.1 and Equation

(5.32) holds because of no contract on the market. $\square$

From Theorem 5.2, we find that the variance of CP is reduced with the introduction of bilateral contracts.

We now consider the situation that the number of gencos goes to infinity. We first explore two lemmas.

Lemma 5.2.

$$\begin{aligned}(i) \quad \lim_{n\to\infty} \frac{b\bar{Q}}{n(a+\epsilon)-\sum_{k=1}^{n} c_k} &= 0. \\ (ii) \quad \lim_{n\to\infty} \frac{b\bar{Q}}{a+\epsilon+\sum_{k=1}^{n} c_k} &= 0.\end{aligned}$$

Proof. (i) We first show that

$$\lim_{n\to\infty} \frac{b\bar{Q}}{n(a+\epsilon)-\sum_{k=1}^{n} c_k} \geq 0. \tag{5.33}$$

Since $Q_s \geq 0$, we have $n(a+\epsilon)-\sum_{k=1}^{n} c_k \geq 0$ from Equation (5.6). Thus, Equation (5.33) holds.

From Assumption 5-2, we have $0 \leq c_i \leq c_{i+1}$, for $i = 1, 2, \ldots, n-1$. Hence, $\sum_{k=1}^{n} c_k \leq nc_n$. Also, because $c_n \leq a+\epsilon-b$, we have $a+\epsilon-c_n \geq b$. Next, we show that

$$\begin{aligned}\lim_{n\to\infty} \frac{b\bar{Q}}{n(a+\epsilon)-\sum_{k=1}^{n} c_k} &\leq \lim_{n\to\infty} \frac{b\bar{Q}}{n(a+\epsilon-c_n)} \\ &\leq \lim_{n\to\infty} \frac{b\bar{Q}}{nb} \\ &= 0,\end{aligned} \tag{5.34}$$

where the first inequality holds from $\sum_{k=1}^{n} c_k \leq nc_n$ and the second inequality from $a+\epsilon-c_n \geq b$.

From Equations (5.33) and (5.34), we have

$$\lim_{n\to\infty} \frac{b\bar{Q}}{n(a+\epsilon)-\sum_{k=1}^{n} c_k} = 0.$$

(ii) We first show that

$$\lim_{n\to\infty} \frac{b\bar{Q}}{a+\epsilon+\sum_{k=1}^{n} c_k} \geq 0, \tag{5.35}$$

where the inequality holds from $b > 0$, $\bar{Q} > 0$ and $a + \epsilon > 0$ and $c_i > 0$, for $i = 1, 2, \ldots, n$.

From Assumption 5-2, we have $0 \leq c_i \leq c_{i+1}$, for $i = 1, 2, \ldots, n-1$. Hence, $\sum_{k=1}^{n} c_k \geq nc_1$. Next, we show that

$$\begin{aligned} \lim_{n\to\infty} \frac{b\bar{Q}}{a + \epsilon + \sum_{k=1}^{n} c_k} &\leq \lim_{n\to\infty} \frac{b\bar{Q}}{a + \epsilon + nc_1} \\ &= 0, \end{aligned} \tag{5.36}$$

where the inequality holds from $\sum_{k=1}^{n} c_k \geq nc_1$.

From Equations (5.35) and (5.36),

$$\lim_{n\to\infty} \frac{b\bar{Q}}{a + \epsilon + \sum_{k=1}^{n} c_k} = 0. \qquad \square$$

Lemma 5.3. *Given* $0 \leq c_i \leq c_{i+1}$*, for* $i = 1, 2, \ldots, n-1$*, and* $\lim_{n\to\infty} c_n = \bar{c}$*, we have*

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} c_i}{n} = K\bar{c},$$

where K *is a constant value in* $[0, 1]$.

Proof. From $0 \leq c_i \leq c_{i+1}$ and $\lim_{n\to\infty} c_n = \bar{c}$, we have

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} c_i}{n} \leq \lim_{n\to\infty} \frac{n\bar{c}}{n} = \bar{c},$$

and

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} c_i}{n} \geq 0.$$

Thus,

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} c_i}{n} = K\bar{c},$$

where K is a constant value in $[0, 1]$. $\square$

When the number of gencos goes to infinity, we show that the MCQ and MCP are unchanged with and without contracts. Moreover, we have the SMQ is a fixed portion of the MCQ without contracts in this situation.

Theorem 5.3.

$$\textit{(i)} \qquad \lim_{n\to\infty} \frac{\hat{Q}_s}{Q_s} = 1.$$

$$\textit{(ii)} \qquad \lim_{n\to\infty} \frac{\hat{P}_s}{P_s} = 1.$$

$$\textit{(iii)} \qquad \lim_{n\to\infty} \frac{\hat{Q}_s - \bar{Q}}{Q_s} = 1 - \frac{b\bar{Q}}{a+\epsilon-K\bar{c}}.$$

Proof. (i) From Lemmas 5.1 and 5.2, as well as Equation (5.6), we have

$$\begin{aligned}
\lim_{n\to\infty} \frac{\hat{Q}_s}{Q_s} &= \lim_{n\to\infty} \frac{Q_s + \frac{\bar{Q}}{(n+1)}}{Q_s} \\
&= \lim_{n\to\infty} 1 + \frac{\bar{Q}}{(n+1)\cdot\left(\frac{n(a+\epsilon)-\sum_{k=1}^{n} c_k}{(n+1)b}\right)} \\
&= 1 + \lim_{n\to\infty} \frac{b\bar{Q}}{n(a+\epsilon)-\sum_{k=1}^{n} c_k} \\
&= 1.
\end{aligned}$$

(ii) From Lemmas 5.1 and 5.2, as well as Equation (5.10), we have

$$\begin{aligned}
\lim_{n\to\infty} \frac{\hat{P}_s}{P_s} &= \lim_{n\to\infty} \frac{P_s - \frac{b\bar{Q}}{(n+1)}}{P_s} \\
&= \lim_{n\to\infty} 1 - \frac{b\bar{Q}}{(n+1)\cdot\left(\frac{a+\epsilon+\sum_{k=1}^{n} c_k}{n+1}\right)} \\
&= 1 - \lim_{n\to\infty} \frac{b\bar{Q}}{a+\epsilon+\sum_{k=1}^{n} c_k} \\
&= 1.
\end{aligned}$$

(iii) From Lemmas 5.1 and 5.3, as well as Equation (5.6), we have

$$\begin{aligned}
\lim_{n\to\infty} \frac{\hat{Q}_s - \bar{Q}}{Q_s} &= \lim_{n\to\infty} \frac{Q_s - \frac{n\bar{Q}}{(n+1)}}{Q_s} \\
&= \lim_{n\to\infty} 1 - \frac{n\bar{Q}}{(n+1)\cdot\left(\frac{n(a+\epsilon)-\sum_{k=1}^{n} c_k}{(n+1)b}\right)} \\
&= 1 - \lim_{n\to\infty} \frac{b\bar{Q}}{(a+\epsilon) - \frac{\sum_{k=1}^{n} c_k}{n}} \\
&= 1 - \frac{b\bar{Q}}{a+\epsilon-K\bar{c}},
\end{aligned}$$

where K is a constant value in $[0, 1]$. □

From Theorem 5.3, we find that the impact of bilateral contracts on MCQ and MCP reduces as the number of gencos increases. When the number of gencos goes to infinity, the impact is minimized. Moreover, we find the closed form ratio for SMQ divided by MCQ without contracts. When contract quantity is very small, SMQ tends to equal to MCQ without contracts. By increasing the contract quantity, the difference between SMQ and MCQ without contracts increases.

Given the total contract quantity, we show that MCQ with contracts, SMQ and MCP are independent from the allocation of the total contract quantity among gencos.

Theorem 5.4. *Consider a fixed $\bar{Q}$. We have $\hat{Q}_s$, $\hat{Q}_s - \bar{Q}$ and $\hat{P}_s$ with no relationship with $\bar{Q}_i$, for $i = 1, 2, \ldots, n$.*

Proof. From Equations (5.17), (5.18) and (5.23), the allocation of the fixed contract quantity, $\bar{Q}$, among gencos has no relationship with MCQ with contracts, SMQ and MCP. □

5.2.4 Market total profit and monopoly ratio

In this section, we show the impact of bilateral contracts on the market total profit. We first discuss the case that gencos have non-identical marginal costs. Then, we discuss the case that gencos have identical marginal cost.

Gencos with non-identical marginal costs

Recall in Assumption 5-2, we have $0 \leq c_i \leq c_{i+1}$, for $i = 1, 2, \ldots, n-1$. We now study the total profit of the market. We first show a closed form of total profit with and without contracts in Theorem 5.5. Then, we observe that low contract price may reduce the total profit of the market in Theorem 5.6.

Theorem 5.5.

$$\sum_{i=1}^{n} \hat{\pi}_n^i(x_i^{**}) = \left(\sum_{i=1}^{n} \pi_n^i(x_i^*)\right) + \sum_{i=1}^{n} \frac{-2\bar{Q}\left[a+\epsilon-(n+1)c_i+\sum_{k=1}^{n} c_k\right]+b\bar{Q}^2}{(n+1)^2} + \sum_{i=1}^{n} \left(\bar{P}-c_i\right)\bar{Q}_i.$$

Proof. From Equations (5.14) and (5.28), we have

$$\begin{aligned}\sum_{i=1}^{n} \hat{\pi}_n^i(x_i^{**}) &= \left(\sum_{i=1}^{n} \frac{[a+\epsilon-(n+1)c_i+(\sum_{k=1}^{n} c_k)-b\bar{Q}]^2}{(n+1)^2 b}\right) + \sum_{i=1}^{n} \left(\bar{P}-c_i\right)\bar{Q}_i \\ &= \left(\sum_{i=1}^{n} \pi_n^i(x_i^*)\right) + \sum_{i=1}^{n} \frac{-2\bar{Q}\left[a+\epsilon-(n+1)c_i+\sum_{k=1}^{n} c_k\right]+b\bar{Q}^2}{(n+1)^2} \\ &\quad + \sum_{i=1}^{n} \left(\bar{P}-c_i\right)\bar{Q}_i. \qquad \square\end{aligned}$$

Theorem 5.6. *For* $\bar{P} \leq c_1$ *and* $x_i^{**} \geq \bar{Q}_i$,

$$\sum_{i=1}^{n} \hat{\pi}_n^i(x_i^{**}) < \sum_{i=1}^{n} \pi_n^i(x_i^*).$$

Proof. For $\bar{P} \leq c_1$, Equations (5.14) and (5.28) imply

$$\begin{aligned}\sum_{i=1}^{n} \hat{\pi}_n^i(x_i^{**}) &= \left(\sum_{i=1}^{n} \frac{[a+\epsilon-(n+1)c_i+(\sum_{k=1}^{n} c_k)-b\bar{Q}]^2}{(n+1)^2 b}\right) + \sum_{i=1}^{n} \left(\bar{P}-c_i\right)\bar{Q}_i \\ &\leq \sum_{i=1}^{n} \frac{[a+\epsilon-(n+1)c_i+(\sum_{k=1}^{n} c_k)-b\bar{Q}]^2}{(n+1)^2 b} \\ &< \sum_{i=1}^{n} \frac{[a+\epsilon-(n+1)c_i+\sum_{k=1}^{n} c_k]^2}{(n+1)^2 b} \qquad (5.37) \\ &= \sum_{i=1}^{n} \pi_n^i(x_i^*),\end{aligned}$$

where Equation (5.37) holds from $\bar{Q} > 0$. $\square$

Gencos with identical marginal cost

In this section, we discuss a special case where all the gencos have the same MC, that is $c_1 = c_2 = \cdots = c_n = c$. We now consider monopoly ratio defined as the total profit divided by the maximal profit when one genco monopolizes the electricity market, that is $\sum_{i=1}^{n} \hat{\pi}_n^i(x^{**})/\hat{\pi}_1^1(x^{**})$. Here, $\hat{\pi}_1^1(x^{**})$ is the profit where only one genco monopolizes the electricity market. Hence, we have

$$\hat{\pi}_1^1(x^{**}) = \frac{(a+\epsilon-c-b\bar{Q})^2}{4b} + (\bar{P}-c)\bar{Q}, \tag{5.38}$$

where Equation (5.38) holds from Equation (5.27).

Let $A = a+\epsilon-c-b\bar{Q}$. From Equation (5.19), we have $\bar{Q} \leq a+\epsilon-nc/b$. As a result, we have $A \geq 0$. For all gencos with the same marginal cost, we have the monopoly ratio as

$$\begin{aligned} \frac{\sum_{i=1}^{n} \hat{\pi}_n^i(x^{**})}{\hat{\pi}_1^1(x^{**})} &= \frac{n(a+\epsilon-c-b\bar{Q})^2/(n+1)^2 b + \left(\bar{P}-c\right)\bar{Q}}{(a+\epsilon-c-b\bar{Q})^2/4b + \left(\bar{P}-c\right)\bar{Q}} \\ &= \frac{4}{(n+1)^2} \cdot \frac{nA^2+(n+1)^2 b(\bar{P}-c)\bar{Q}}{A^2+4b(\bar{P}-c)\bar{Q}}, \end{aligned} \tag{5.39}$$

where the first equation holds from Equations (5.28) and (5.38).

We now show that the monopoly ratio is a decreasing function of the number of gencos.

Theorem 5.7. *For all gencos with the same marginal cost, the monopoly ratio is a decreasing function of n.*

Proof. Taking the first partial derivative of $\sum_{i=1}^{n} \hat{\pi}_n^i(x^{**})/\hat{\pi}_1^1(x^{**})$ with respect to n, we have from Equation (5.39)

$$\frac{\partial[\sum_{i=1}^{n} \hat{\pi}_n^i(x^{**})/\hat{\pi}_1^1(x^{**})]}{\partial n} = \frac{4A^2(1-n)}{\left[A^2+4b(\bar{P}-c)\bar{Q}\right](n+1)^3} \leq 0,$$

where the inequality holds from net profit of genco i greater than 0. Hence, we have the monopoly ratio as a decreasing function of n. $\square$

From Theorem 5.7, we find that more gencos reduce the monopoly ratio. This observation is reasonable since more gencos usually lead to a more competitive market.

We now show that the monopoly ratio is an increasing function of contract price.

Theorem 5.8. *For all gencos with the same marginal cost, the monopoly ratio is an increasing function of $\bar{P}$.*

Proof. Taking the first partial derivative of $\sum_{i=1}^{n} \hat{\pi}_n^i(x^{**})/\hat{\pi}_1^1(x^{**})$ with respect to $\bar{P}$, we have from Equation (5.39)

$$\begin{aligned}
\frac{\partial \left[\sum_{i=1}^{n} \hat{\pi}_n^i(x^{**})/\hat{\pi}_1^1(x^{**})\right]}{\partial \bar{P}} &= \frac{4}{(n+1)^2} \cdot \frac{1}{\left[A^2 + 4b\left(\bar{P} - c\right)\bar{Q}\right]^2} \\
&\quad \cdot \{(n+1)^2 b\bar{Q}[A^2 + 4b\left(\bar{P} - c\right)\bar{Q}] \\
&\qquad -4b\bar{Q}[nA^2 + (n+1)^2 b\left(\bar{P} - c\right)\bar{Q}]\} \\
&= \frac{4}{(n+1)^2} \cdot \frac{(n-1)^2 b\bar{Q}A^2}{\left[A^2 + 4b\left(\bar{P} - c\right)\bar{Q}\right]^2} \\
&\geq 0,
\end{aligned}$$

where the inequality holds from net profit of genco i greater than 0. Hence, we have the monopoly ratio as an increasing function of contract price, $\bar{P}$. □

From Theorem 5.8, we find that higher contract price increases the total profit of all gencos. On the other hand, customers suffer from the high contract price. Hence, the contract price should be set carefully.

We now show that the monopoly ratio is an increasing function of contract quantity if all gencos have the same MC and MC is less than the contract price. Otherwise, the monopoly ratio is a decreasing function of contract quantity.

Theorem 5.9. *If all gencos have the same marginal cost, c, and $\bar{P} > c$, the monopoly ratio is an increasing function of $\bar{Q}$. Otherwise, the monopoly ratio is a decreasing function of $\bar{Q}$.*

Proof. Taking the first partial derivative of $\sum_{i=1}^{n} \hat{\pi}_n^i(x^{**})/\hat{\pi}_1^1(x^{**})$ with respect to

$\bar{Q}$, we have from Equation (5.39)

$$\begin{aligned}
\frac{\partial\left[\sum_{i=1}^{n}\hat{\pi}_n^i(x^{**})/\hat{\pi}_1^1(x^{**})\right]}{\partial\bar{Q}} &= \frac{4}{(n+1)^2}\cdot\frac{b}{\left[A^2+4b(\bar{P}-c)\bar{Q}\right]^2}\\
&\quad\cdot\{[-2nA+(n+1)^2(\bar{P}-c)][A^2+4b\left(\bar{P}-c\right)\bar{Q}]\\
&\qquad -[-2(A+4(\bar{P}-c)][nA^2+(n+1)^2b\left(\bar{P}-c\right)\bar{Q}]\}\\
&= \frac{4}{(n+1)^2}\cdot\frac{(n-1)^2b(\bar{P}-c)[A^2+2b\bar{Q}A]}{\left[A^2+4b\left(\bar{P}-c\right)\bar{Q}\right]^2}.
\end{aligned}$$

Since $\bar{Q}\in[0,(a+\epsilon-c)/b]$, we have $\sum_{i=1}^{n}\hat{\pi}_n^i(x^{**})/\hat{\pi}_1^1(x^{**})$ is an increasing function of contract quantity for $\bar{P}>c$. When $\bar{P}\le c$, we have the monopoly ratio is a decreasing function of contract quantity. $\square$

5.2.5 Market power

In this section, we investigate the market power, which is defined as the ability of a genco to maximize its own profit by using bidding strategies (Wolak, 2000). Generally, the market power is measured by a popular market power index, called the Lerner Index.

Lerner Index

The Lerner Index is defined as (Chang, 2007)

$$\text{Lerner Index} = \frac{\text{market clearing price} - \text{marginal cost}}{\text{market clearing price}}.$$

The smaller the Lerner Index, the less the market power. Moreover, the Lerner Index is an increasing function of MCP. Without contracts, the Lerner Index for genco i is

$$\mathcal{L}_i=\frac{P_s-c_i}{P_s}.$$

Thus, when there are bilateral contracts on the market, the Lerner Index can be rewritten as

$$\mathcal{L}_i=\frac{\hat{P}_s-c_i}{\hat{P}_s}. \tag{5.40}$$

We now show the impact of contract price and contract quantity on the Lerner Index.

Theorem 5.10. *$\mathcal{L}_i$ is a constant function of $\bar{P}$, for $i = 1, 2, \ldots, n$.*

Proof. From Equation (5.23), the MCP with contracts is a constant function of contract price. From Equation (5.40), the Lerner Index has no relationship with the contract price. □

Theorem 5.11. *$\mathcal{L}_i$ is a decreasing function of $\bar{Q}$, for $i = 1, 2, \ldots, n$.*

Proof. From Equation (5.23), the MCP is a decreasing function of contract quantity. Since the Lerner Index is an increasing function of MCP, we have the Lerner Index as a decreasing function of contract quantity from Equation (5.40). □

Theorems 5.10 and 5.11 show the impact of contract price and contract quantity on the Lerner Index. That is, the Lerner Index is a constant function of contract price. Moreover, the Lerner Index is a decreasing function of contract quantity.

Profit Index

We propose a new index called the Profit Index, to evaluate the market power. The Profit Index is defined as the ratio of profit with and without competition. In this section, we first discuss the case that gencos have non-identical marginal costs. Then, we discuss the case that gencos have identical marginal cost.

Gencos with non-identical marginal costs Define the Profit Index of genco i without contracts as

$$\eta_i = \frac{\pi_n^i(x^*)}{\pi_1^i(x^*)},$$

where $\pi_n^i(x^*)$ is the profit of genco i without contracts in the n-genco market and $\pi_1^i(x^*)$ is the profit of genco i without contracts in the single genco market. From

Equation (5.14), the Profit Index of genco i without contracts is

$$\begin{aligned}\eta_i &= \frac{\pi_n^i(x^*)}{\pi_1^i(x^*)} \\ &= \left(\frac{[a+\epsilon-(n+1)c_i+\sum_{k=1}^n c_k]^2}{(n+1)^2 b}\right) \Bigg/ \left(\frac{(a+\epsilon-c_i)^2}{4b}\right) \\ &= \frac{4[(a+\epsilon-(n+1)c_i+\sum_{k=1}^n c_k)^2]}{(n+1)^2(a+\epsilon-c_i)^2}.\end{aligned}$$

Thus, when there are bilateral contracts on the market, the Profit Index can be rewritten as

$$\eta_i = \frac{\hat{\pi}_n^i(x^{**})}{\hat{\pi}_1^i(x^{**})}, \tag{5.41}$$

where $\hat{\pi}_n^i(x^{**})$ is the profit of genco i with contracts in the n-genco market and $\hat{\pi}_1^i(x^{**})$ is the profit of genco i with contracts in the single genco market.

We now investigate the impact of contract price on the Profit Index. The profit of genco i with contract quantity $\bar{Q}_i$ and contract price $\bar{P}$ in the single genco market is

$$\hat{\pi}_1^i(x^{**}) = \frac{(a+\epsilon-c_i-b\bar{Q}_i)^2}{4b} + (\bar{P}-c_i)\bar{Q}_i, \tag{5.42}$$

where Equation (5.42) holds from Equation (5.27). Thus, the Profit Index of genco i with contracts is

$$\begin{aligned}\eta_i &= \frac{\hat{\pi}_n^i(x^{**})}{\hat{\pi}_1^i(x^{**})} \\ &= \frac{\left[a+\epsilon-(n+1)c_i+(\sum_{k=1}^n c_k)-b\bar{Q}\right]^2/[(n+1)^2 b]+(\bar{P}-c_i)\bar{Q}_i}{(a+\epsilon-c_i-b\bar{Q}_i)^2/(4b)+(\bar{P}-c_i)\bar{Q}_i}. \end{aligned} \tag{5.43}$$

Now, we show that the Profit Index of genco i is an increasing function of contract price.

Lemma 5.4. *For* $x_i^{**} \geq \bar{Q}_i$,

$$\hat{\pi}_1^i(x^{**}) \geq \hat{\pi}_n^i(x^{**}).$$

Proof. Let $B = a + \epsilon - b\bar{Q}$. From Equation (5.20), for $x_i^{**} \geq \bar{Q}_i$, we have

$$\begin{aligned} x_j^{**} - \bar{Q}_j &= \frac{a + \epsilon - (n+1)c_j + (\sum_{k=1}^n c_k) + (n+1)b\bar{Q}_j - b\bar{Q}}{(n+1)b} - \bar{Q}_j \\ &= \frac{a + \epsilon - (n+1)c_j + (\sum_{k=1}^n c_k) - b\bar{Q}}{(n+1)b} \\ &= \frac{B - nc_j + [(\sum_{k=1}^n c_k) - c_j]}{(n+1)b} \\ &\geq 0, \qquad\qquad \text{for} \quad j = 1, 2, \ldots, n. \end{aligned}$$

From the above inequality, we have

$$c_j \leq \frac{B + [(\sum_{k=1}^n c_k) - c_j]}{n}.$$

Thus,

$$\sum_{k \neq i}^n c_k \leq \frac{(n-1)B + (n-1)c_i + (n-2)\sum_{k\neq i}^n c_k}{n}.$$

From the above inequality, we can simplify it as

$$\sum_{k \neq i}^n c_k \leq \frac{(n-1)(B + c_i)}{2}. \tag{5.44}$$

Hence,

$$\begin{aligned} &\frac{B - c_i}{2} - \frac{B - (n+1)c_i + \sum_{k=1}^n c_k}{n+1} \\ &\qquad = \frac{(n+1)(B - c_i) - 2[B - (n+1)c_i + \sum_{k=1}^n c_k]}{2(n+1)} \\ &\qquad = \frac{(n-1)B + (n+1)c_i - 2\sum_{k=1}^n c_k}{2(n+1)} \\ &\qquad = \frac{(n-1)B + (n-1)c_i - 2\sum_{k\neq i}^n c_k}{2(n+1)} \\ &\qquad = \frac{(n-1)(B + c_i) - 2\sum_{k\neq i}^n c_k}{2(n+1)} \\ &\qquad \geq 0, \end{aligned} \tag{5.45}$$

where Equation (5.45) holds from Equation (5.44).

From Equation (5.19), we have

$$\bar{Q} \leq \frac{a + \epsilon - (n+1)c_n + \sum_{k=1}^n c_k}{b}.$$

Then,

$$a + \epsilon - (n+1)c_n + \sum_{k=1}^{n} c_k - b\bar{Q} \geq 0.$$

Since $0 \leq c_i \leq c_{i+1}$, for $i = 1, 2, \ldots, n-1$, we have

$$a + \epsilon - c_n - b\bar{Q} \geq nc_n - \sum_{k=1}^{n} c_k \geq 0.$$

Hence,

$$a + \epsilon - c_i - b\bar{Q} \geq 0. \tag{5.46}$$

From Equation (5.27), we have

$$\begin{aligned}
\hat{\pi}_1^i(x^{**}) - \hat{\pi}_n^i(x^{**}) &= \frac{(a+\epsilon-c_i-b\bar{Q}_i)^2}{4b} + (\bar{P}-c_i)\bar{Q}_i \\
&\quad - \left(\frac{\left[a+\epsilon-(n+1)c_i+(\sum_{k=1}^n c_k)-b\bar{Q}\right]^2}{(n+1)^2 b} + (\bar{P}-c_i)\bar{Q}_i\right) \\
&= \frac{(a+\epsilon-c_i-b\bar{Q}_i)^2}{4b} - \frac{\left[a+\epsilon-(n+1)c_i+(\sum_{k=1}^n c_k)-b\bar{Q}\right]^2}{(n+1)^2 b} \\
&\geq \frac{(a+\epsilon-c_i-b\bar{Q})^2}{4b} - \frac{\left[a+\epsilon-(n+1)c_i+(\sum_{k=1}^n c_k)-b\bar{Q}\right]^2}{(n+1)^2 b} \qquad (5.47) \\
&= \frac{1}{b}\left(\frac{(B-c_i)^2}{4} - \frac{\left[B-(n+1)c_i+\sum_{k=1}^n c_k\right]^2}{(n+1)^2}\right) \\
&= \frac{1}{b}\left(\frac{B-c_i}{2} + \frac{B-(n+1)c_i+\sum_{k=1}^n c_k}{n+1}\right) \\
&\quad \cdot \left(\frac{B-c_i}{2} - \frac{B-(n+1)c_i+\sum_{k=1}^n c_k}{n+1}\right) \\
&\geq 0, \qquad (5.48)
\end{aligned}$$

where Equation (5.47) holds from $\bar{Q}_i \leq \bar{Q}$ and Equation (5.46). Moreover, Equation (5.48) holds from Equation (5.45). □

Theorem 5.12. *For $x_i^{**} \geq \bar{Q}_i$, η_i is an increasing function of $\bar{P}$, for $i = 1, 2, \ldots, n$.*

Proof. Taking the first partial derivative of η_i with respect to $\bar{P}$, we have

$$\frac{\partial \eta_i}{\partial \bar{P}} = \frac{\bar{Q}_i[\hat{\pi}_1^i(x^{**}) - \hat{\pi}_n^i(x^{**})]}{c_i \hat{\pi}_1^i(x^{**})^2} \geq 0,$$

where the inequality holds from Lemma 5.4. $\square$

From Theorem 5.10, we find that market power measured by the Lerner Index has no relationship with the contract price. However, market power measured by the Profit Index has a positive relationship with the contract price. This shows that the Lerner Index fails to show the impact of contract price. On the other hand, we find that higher contract price may increase the gencos' profits when bilateral contract is applied. As a result, the proposed Profit Index successfully captures the impact. One possible reason is that contract price has no impact on the MCP. However, the high contract price results in high profit to gencos from bilateral contracts.

Gencos with identical marginal cost In this section, we discuss a special case where all the gencos have the same MC, that is $c_1 = c_2 = ... = c_n = c$. For all gencos with the same marginal cost, we have the Profit Index of genco i with contracts is

$$\begin{aligned}
\eta_i &= \frac{\hat{\pi}_n^i(x^{**})}{\hat{\pi}_1^i(x^{**})} \\
&= \frac{\left[a + \epsilon - (n+1)c_i + (\sum_{k=1}^n c_k) - b\bar{Q}\right]^2 / [(n+1)^2 b] + (\bar{P} - c_i)\bar{Q}_i}{(a + \epsilon - c_i - b\bar{Q}_i)^2/(4b) + (\bar{P} - c_i)\bar{Q}_i} \\
&= \frac{\left[a + \epsilon - c - b\bar{Q}\right]^2 / [(n+1)^2 b] + (\bar{P} - c)\bar{Q}_i}{(a + \epsilon - c - b\bar{Q}_i)^2/(4b) + (\bar{P} - c)\bar{Q}_i},
\end{aligned} \tag{5.49}$$

where the second equation holds from Equation (5.27).

We now show that the Profit Index is a decreasing function of the number of gencos.

Theorem 5.13. *For all gencos with the same marginal cost, the Profit Index is a decreasing function of n.*

Proof. Taking the first partial derivative of η_i with respect to n, from Equation

(5.49) we have

$$\begin{aligned} \frac{\partial \eta_i}{\partial n} &= -\frac{2\left[a+\epsilon-c-b\bar{Q}\right]^2/[(n+1)^3 b]}{(a+\epsilon-c-b\bar{Q}_i)^2/(4b)+(\bar{P}-c)\bar{Q}_i} \\ &\leq 0, \end{aligned}$$

where the inequality holds from net profit of genco i greater than 0. Hence, the Profit Index is a decreasing function of n. □

From Theorem 5.13, we find that more gencos reduces the Profit Index. This observation is reasonable since more gencos usually lead to a more competitive market.

Gencos with quadratic production cost In the case that gencos with quadratic production cost, we first examine the relationship between profit of a genco and number of gencos. From Equation (4.21), we have

$$\begin{aligned} \frac{\partial \hat{P}_s}{\partial n} &= \left(\frac{a+abc-\bar{Q}}{b}\right)\left(-\frac{1}{n+1+bc}\right) \\ &= \left(\frac{(1+bc)[a-\bar{Q}/(1+bc)]}{b}\right)\left(-\frac{1}{n+1+bc}\right) \\ &= \left(-\frac{1+bc}{b}\right)\left(\frac{a-\bar{Q}/(1+bc)}{n+1+bc}\right). \end{aligned} \tag{5.50}$$

From Equation (4.17), we have

$$\begin{aligned} \frac{\partial x_i^{**}}{\partial n} &= \frac{a-\bar{Q}/(1+bc)}{(n+1+bc)^2} \\ &= \left(\frac{1}{n+1+bc}\right)\left(\frac{a-\bar{Q}/(1+bc)}{n+1+bc}\right). \end{aligned} \tag{5.51}$$

Then, from Equations (5.50) and (5.51), we have

$$
\begin{aligned}
\frac{\partial \hat{\pi}^i(x_i^{**})}{\partial n} &= \frac{\partial(\hat{P}_s(x_i^{**} - \bar{Q}_i) + \bar{P}\bar{Q}_i - P_C(x_i))}{\partial n} \\
&= \hat{P}_s \frac{\partial x_i^{**}}{\partial n} + (x_i^{**} - \bar{Q}_i)\frac{\hat{P}_s}{\partial n} - c x_i^{**}\frac{\partial x_i^{**}}{\partial n} \\
&= \left(\frac{a - \bar{Q}/(1+bc)}{n+1+bc}\right) \\
&\quad \cdot \left[\hat{P}_s\left(\frac{1}{n+1+bc}\right) + (x_i^{**} - \bar{Q}_i)\left(-\frac{1+bc}{b}\right) - c x_i^{**}\left(\frac{1}{n+1+bc}\right)\right] \\
&= \left(\frac{a - \bar{Q}/(1+bc)}{n+1+bc}\right)\left(\frac{1}{b(n+1+bc)}\right) \\
&\quad \cdot \left(b\hat{P}_s - (x_i^{**} - \bar{Q}_i)(n+1+bc)(1+bc) - bc x_i^{**}\right) \\
&= \left(\frac{a - \bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
&\quad \cdot \left(b\hat{P}_s + \bar{Q}_i(n+1+bc)(1+bc) - [(n+1+bc)(1+bc) + bc]x_i^{**}\right) \\
&= \left(\frac{a - \bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
&\quad \cdot \left[\frac{a(1+bc) - \bar{Q}}{n+1+bc} + \bar{Q}_i(n+1+bc)(1+bc)\right. \\
&\quad -[(n+1+bc)(1+bc) + bc] \\
&\quad \left. \cdot \left(\frac{a + \left((n+1+bc)\bar{Q}_i - \bar{Q}\right)/(1+bc)}{n+1+bc}\right)\right] \qquad (5.52) \\
&= \left(\frac{a - \bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
&\quad \cdot \left[\frac{a(1+bc) - \bar{Q}}{n+1+bc} + \bar{Q}_i(n+1+bc)(1+bc)\right. \\
&\quad -a(1+bc) - (n+1+bc)\bar{Q}_i + \bar{Q} \\
&\quad \left. -bc\left(\frac{a + \left((n+1+bc)\bar{Q}_i - \bar{Q}\right)/(1+bc)}{n+1+bc}\right)\right] \\
&= \left(\frac{a - \bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
&\quad \cdot \left[\frac{a(1+bc) - \bar{Q}}{n+1+bc} - a(1+bc) + \bar{Q}\right. \\
&\quad \left. -bc\left(\frac{a + \left((n+1+bc)\bar{Q}_i - \bar{Q}\right)/(1+bc)}{n+1+bc} - \bar{Q}_i(n+1+bc)\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \left(\frac{a-\bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
& \cdot\left[-\frac{[a(1+bc)-\bar{Q}](n+bc)}{n+1+bc}\right. \\
& \left.-bc\left(\frac{a+\left((n+1+bc)\bar{Q}_i-\bar{Q}\right)/(1+bc)}{n+1+bc}-\bar{Q}_i(n+1+bc)\right)\right] \\
= \quad & \left(\frac{a-\bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
& \cdot\left[-\frac{[a(1+bc)-\bar{Q}](n+bc)}{n+1+bc}+bc\bar{Q}_i(n+bc)\right. \\
& \left.-bc\left(\frac{a+\left((n+1+bc)\bar{Q}_i-\bar{Q}\right)/(1+bc)}{n+1+bc}-\bar{Q}_i\right)\right] \\
= \quad & \left(\frac{a-\bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
& \cdot\left[-\left(\frac{n+bc}{n+1+bc}\right)[a(1+bc)-\bar{Q}-bc(n+1+bc)\bar{Q}_i]\right. \\
& \left.-bc\left(\frac{a+\left((n+1+bc)\bar{Q}_i-\bar{Q}\right)/(1+bc)}{n+1+bc}-\bar{Q}_i\right)\right] \\
= \quad & \left(\frac{a-\bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
& \cdot\left[-\left(\frac{n+bc}{n+1+bc}\right)[a(1+bc)-\bar{Q}+(n+1+bc)\bar{Q}_i-(1+bc)(n+1+bc)\bar{Q}_i]\right. \\
& \left.-bc\left(\frac{a+\left((n+1+bc)\bar{Q}_i-\bar{Q}\right)/(1+bc)}{n+1+bc}-\bar{Q}_i\right)\right] \\
= \quad & \left(\frac{a-\bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
& \cdot\left[-(n+bc)(1+bc)\left(\frac{a+[(n+1+bc)\bar{Q}_i-\bar{Q}]/(1+bc)}{n+1+bc}-\bar{Q}_i\right)\right. \\
& \left.-bc\left(\frac{a+\left((n+1+bc)\bar{Q}_i-\bar{Q}\right)/(1+bc)}{n+1+bc}-\bar{Q}_i\right)\right] \\
= \quad & \left(\frac{a-\bar{Q}/(1+bc)}{b(n+1+bc)^2}\right) \\
& \cdot[-(n+bc)(1+bc)(x_i^{**}-\bar{Q}_i)-bc(x_i^{**}-\bar{Q}_i)] \qquad (5.53) \\
= \quad & -\left(\frac{a-\bar{Q}/(1+bc)}{b(n+1+bc)^2}\right)[(n+bc)(1+bc)+bc](x_i^{**}-\bar{Q}_i), \qquad (5.54)
\end{aligned}
$$

where Equation (5.52) holds from Equations (4.17) and (4.21), and Equation (5.53) holds from Equation (4.17). For $x_i^{**} \geq \bar{Q}_i$, we have $\partial\hat{\pi}^i(x_i^{**})/\partial n < 0$ from (5.54).

Hence, Lemma 5.4 still holds.

Taking the first partial derivative of η_i with respect to $\bar{P}$, we have

$$\frac{\partial \eta_i}{\partial \bar{P}} = \frac{\bar{Q}_i[\hat{\pi}_1^i(x^{**}) - \hat{\pi}_n^i(x^{**})]}{c\hat{\pi}_1^i(x^{**})^2} \geq 0,$$

where the inequality holds from Lemma 5.4. Hence, Theorem 5.12 still holds in the case that genecos with quadratic production cost.

Taking the first partial derivative of η_i with respect to n, we have

$$\frac{\partial \eta_i}{\partial n} = \frac{\partial \left(\frac{\hat{\pi}_n^i(x^{**})}{\hat{\pi}_1^i(x^{**})} \right)}{\partial n} = \frac{\frac{\partial \hat{\pi}_n^i(x^{**})}{\partial n}}{\hat{\pi}_1^i(x^{**})} \leq 0,$$

where the inequality holds from Lemma 5.4. Hence, Theorem 5.13 still holds in the case that genecos with quadratic production cost.

5.3 Numerical study

In this section, we verified our new proposed Profit Index and the relative two theorems: Theorems 5.12 and 5.13. Due to the length of the thesis, we will not implement the numerical study for Lerner Index. More information about Lerner Index can be seen in Chang (2007).

In Singapore electricity market, demand quantity is forecasted as a constant value which is independent of price. It can be interpreted as a special case of a linear demand function. Besides, the assumption of quadratic production cost is verified in Chapter 4. We now use the data of Singapore electricity market from 2004 to 2010 to verify Theorems 5.12 and 5.13 under the environment of linear demand function and quadratic production cost.

We use the same data of hedge price, hedge ratio, USEP and demand quantity as Chapters 3 and 4. We also use the data of annual market shares for different gencos from 2004 to 2010 which are available in the website of Energy Market

Company. Based on the annual market shares, the numbers of gencos from 2004 to 2010 can be estimated (see Table 5.3). Besides, the data of crude oil prices are collected from the website of U.S. Energy Information Administration.

Table 5.3: Estimated number of gencos and annual market shares by gencos

Year	Estimated number of gencos	Gencos					
		Senoko Power	PowerSeraya	Tuas Power	SembCorp	Keppel	Others
2004	4	33%	28%	23%	13%	0%	3%
2005	4	32%	29%	24%	12%	0%	3%
2006	4	32%	28%	26%	11%	0%	2%
2007	5	30%	28%	25%	9%	6%	2%
2008	5	28%	25%	24%	11%	10%	2%
2009	5	26%	27%	24%	11%	9%	3%
2010	5	26%	28%	25%	10%	8%	3%

We analyze the data collected from 2004 to 2010 and there are 28 quarters. Different hedge prices and hedge quantities are assigned by the Singapore government. In each quarter, the Singapore government decides one hedge price and equally partitions the periods into three types: “Peak”, “Shoulder” and “OffPeak”, where the “Peak” periods are assigned with the highest hedge quantity and the “OffPeak” periods are assigned with the lowest hedge quantity (Energy Market Authority (EMA) Singapore, 2010b). Thus, the period numbers of the three types are nearly equal. All periods in the same type are assigned with the identical hedge price and quantity. As a result, we consider the data from the same period type in a quarter as an independent hedge condition and generate one Profit Index for the hedge condition.

In our numerical study, we assume that all gencos have the same production cost. There are three reasons for this assumption. One reason is that 97% of the generating units in Singapore rely on fuel oil and natural gas to generate electricity (Singapore Market Surveillance and Compliance Panel (MSCP) Singapore 2007). Therefore, their fuel costs are similar. The second reason is that most of gencos in Singapore use the same technology, combined-cycle gas turbine technology. Moreover, most of the generating units which use this technology are built during the early 2000s (see the website of Energy Market Authority and the website of E^2 Singapore). Hence, they have similar energy efficiency. The third reason is that gencos are equally profitable in the long run. Although the production cost of each genco is confidential and not available to the public. We believe that their production costs are similar and highly related. Under this assumption, the market powers of different gencos are similar.

In our numerical study, we estimate the average production cost as 75% of the mean CP for each hedge condition. There are three reasons for this estimation. The first reason is that the average production costs and the CPs are highly related. Because 97% of the generating units rely on fuel oil and natural gas to generate electricity as mentioned above. The second reason is that the average production cost is usually less than the CP in a regular market. Otherwise, the gencos are losing money which is unreasonable in the long run. The third reason is that for 90% of the total periods, MCPs are no less than 75% of the mean CP. We also raise the estimated average production cost to 80% of the mean CP. However, for only 81% of the total periods, MCPs are no less than 80% of the mean CP. This percentage dramatically decreases as the estimated average production cost increases. Therefore, we use 75% of the mean CP as the estimation of average production cost.

To consider regular and profitable markets, we do not select the periods in which the MCPs are less than the average production cost. As that, we estimate the average production cost as 75% of the mean CP, 90% of the periods are considered. However, the selected data for Quarter 4 of 2008 and Quarter 1 of 2009 are much less (see the eighth column of Tables 5.5, 5.6 and 5.7). The reason may be that the crude oil prices drastically decrease in these two quarters (see the third column of Tables 5.5, 5.6 and 5.7). From the sixth column of Tables 5.5, 5.6 and 5.7, we observe high hedge prices in those two quarters. Thus, the major profits are contributed by the bilateral contracts and gencos can sacrifice profit from the spot market for these two quarters. That is, gencos can accept MCPs less than average production cost. As a result, fewer periods are considered for these two quarters in this numerical study.

In order to calculate the Profit Index, we need to estimate profit for one-genco market. To do so, we first estimate USEP for one-genco market. Thus, we generalize the relationship between USEP and demand in Section 4.3.1. We estimate the USEP for one-genco market by the regression on the nonlinear terms of demand, oil prices and number of gencos. The Mean Absolute Percentage Error (MAPE) of this estimation is 3% only. The annual mean of demand, oil price and demand as well as the estimated number of gencos and USEP for one-genco market from 2004 to 2010 are presented in Table 5.4.

Table 5.4: Estimated USEP for one-genco market

Year	Estimated number of gencos	Mean of Demand	Mean of Oil Price	Mean of USEP	Mean of estimated USEP for one-genco market
2004	4	4042	34	82	206
2005	4	4218	48	110	275
2006	4	4358	58	132	331
2007	5	4547	66	125	374
2008	5	4588	93	162	487
2009	5	4604	72	148	443
2010	5	5009	72	171	512

In this section, we first investigate the relationship between hedge price ratio and Profit Index. Then, we study the relationship between the number of gencos and Profit Index.

5.3.1 Relationship between hedge price ratio and Profit Index

Based on the data of USEP, demand, hedge ratio, estimated average production cost, number of gencos and USEP for one-genco makert, we can calculate the Profit Index for each period by using Equation (5.41). The quarterly mean of Profit Index for "Peak", "Shoulder" and "OffPeak" periods are presented in the tenth column of Tables 5.5, 5.6 and 5.7. Since the average costs for different hedge conditions are different, we use hedge price ratio defined as hedge price divided by average production cost to represent the unified hedge price (see the seventh and ninth columns of Tables 5.5, 5.6 and 5.7).

The correlation coefficients between hedge price ratio and Profit Index for each year are summarized in Table 5.8. We observe that the correlation coefficient for 2007 is not reasonable. The reason may be that the average hedge ratio is reduced from 65% to 55% during 2007. From the fifth column of Tables 5.5, 5.6 and 5.7, we observe that the hedge ratios for Quarter 1 and Quarter 2 of 2007 are higher than the hedge ratios for Quarter 3 and Quarter 4 of 2007.

Among the 21 scenarios, we observe that 19 scenarios have positive correlation coefficients (see Table 5.8). Moreover, the correlation coefficients for 18 scenarios are close to 1.00 (see Table 5.8). Hence, Profit Index has highly positive correlation with hedge price. That is, Profit Index is an increasing function of hedge price. This result supports Theorem 5.12.

5.3.2 Relationship between number of gencos and Profit Index

The number of gencos and annual mean of Profit Index are presented in Table 5.9. The correlation coefficients between number of gencos and Profit Index are summarized in Table 5.10. We observe that the correlation coefficients for "Peak", "Shoulder" and "OffPeak" periods all have negative signs (see Table 5.10). Moreover, the correlation coefficients are close to -1.00. Hence, Profit Index has highly negative correlation with the number of gencos. That is, Profit Index is a decreasing function of the number of gencos. This result supports Theorem 5.13.

Table 5.5: Mean of Profit Index for selected Peak periods

Year	Quarter	Crude oil prices ($/Barrel)	Mean of customer price	Hedge ratio	Hedge price (S$/MWh)	Average cost (S$/MWh)	Selected data	Hedge price ratio	Mean of Profit Index
2004	Q1	29.13	88.12	0.72	94.24	66.09	95.9 %	1.43	0.41
	Q2	32.67	93.13	0.72	96.25	69.85	100.0 %	1.38	0.39
	Q3	36.45	93.16	0.72	95.73	69.87	100.0 %	1.37	0.39
	Q4	38.45	101.42	0.72	101.56	76.07	99.3 %	1.34	0.37
2005	Q1	40.29	97.02	0.75	101.29	72.76	98.7 %	1.39	0.43
	Q2	45.21	99.96	0.75	96.35	74.97	100.0 %	1.29	0.37
	Q3	55.19	121.78	0.75	117.38	91.33	93.9 %	1.29	0.37
	Q4	50.91	129.51	0.75	128.39	97.14	100.0 %	1.32	0.39
2006	Q1	54.52	133.67	0.75	140.70	100.25	94.3 %	1.40	0.44
	Q2	62.90	142.13	0.75	139.44	106.59	100.0 %	1.31	0.38
	Q3	63.51	147.15	0.75	147.90	110.36	98.4 %	1.34	0.40
	Q4	52.71	145.89	0.75	150.20	109.42	92.8 %	1.37	0.42
2007	Q1	51.86	132.63	0.75	134.66	99.47	87.8 %	1.35	0.34
	Q2	61.51	121.91	0.75	121.14	91.43	100.0 %	1.32	0.33
	Q3	69.60	135.25	0.63	137.25	101.44	100.0 %	1.35	0.26
	Q4	82.20	145.71	0.63	150.04	109.28	100.0 %	1.37	0.26
2008	Q1	90.21	158.42	0.63	161.80	118.82	100.0 %	1.36	0.26
	Q2	114.51	182.41	0.63	174.44	136.81	100.0 %	1.28	0.23
	Q3	113.33	192.08	0.63	183.25	144.06	100.0 %	1.27	0.24
	Q4	54.17	195.80	0.63	238.64	146.85	33.9 %	1.63	0.35
2009	Q1	40.16	139.02	0.63	167.14	104.27	13.5 %	1.60	0.34
	Q2	55.84	167.65	0.63	115.26	125.74	100.0 %	0.92	0.15
	Q3	72.46	154.84	0.63	138.92	116.13	99.7 %	1.20	0.20
	Q4	71.67	171.41	0.63	161.70	128.56	100.0 %	1.26	0.22
2010	Q1	74.76	196.41	0.63	171.05	147.31	98.1 %	1.16	0.19
	Q2	74.79	181.40	0.63	176.10	136.05	100.0 %	1.29	0.24
	Q3	72.46	171.28	0.63	176.29	128.46	96.8 %	1.37	0.26
	Q4	80.52	177.53	0.63	165.71	133.15	98.6 %	1.24	0.22

Table 5.6: Mean of Profit Index for selected Shoulder periods

Year	Quarter	Crude oil prices ($/Barrel)	Mean of customer price	Hedge ratio	Hedge price (S$/MWh)	Average cost (S$/MWh)	Selected data	Hedge price ratio	Mean of Profit Index
2004	Q1	29.13	86.12	0.65	94.24	64.59	87.6 %	1.46	0.37
	Q2	32.67	94.73	0.65	96.25	71.05	100.0 %	1.35	0.32
	Q3	36.45	91.27	0.65	95.73	68.45	99.9 %	1.40	0.34
	Q4	38.45	99.60	0.65	101.56	74.70	96.7 %	1.36	0.32
2005	Q1	40.29	98.41	0.65	101.29	73.81	89.0 %	1.37	0.33
	Q2	45.21	100.72	0.65	96.35	75.54	100.0 %	1.28	0.30
	Q3	55.19	115.29	0.65	117.38	86.47	96.7 %	1.36	0.33
	Q4	50.91	128.09	0.65	128.39	96.07	99.9 %	1.34	0.32
2006	Q1	54.52	132.42	0.65	140.70	99.31	88.2 %	1.42	0.35
	Q2	62.90	138.13	0.65	139.44	103.59	100.0 %	1.35	0.32
	Q3	63.51	157.04	0.65	147.90	117.78	79.8 %	1.26	0.28
	Q4	52.71	144.25	0.65	150.20	108.19	74.0 %	1.39	0.34
2007	Q1	51.86	138.83	0.65	134.66	104.12	77.1 %	1.29	0.24
	Q2	61.51	118.57	0.65	121.14	88.93	100.0 %	1.36	0.27
	Q3	69.60	132.65	0.55	137.25	99.49	100.0 %	1.38	0.22
	Q4	82.20	144.55	0.55	150.04	108.41	100.0 %	1.38	0.22
2008	Q1	90.21	159.32	0.55	161.80	119.49	100.0 %	1.35	0.22
	Q2	114.51	180.92	0.55	174.44	135.69	100.0 %	1.29	0.20
	Q3	113.33	187.32	0.55	183.25	140.49	100.0 %	1.30	0.21
	Q4	54.17	183.49	0.55	238.64	137.62	34.3 %	1.73	0.31
2009	Q1	40.16	132.59	0.55	167.14	99.44	12.5 %	1.68	0.30
	Q2	55.84	129.80	0.55	115.26	97.35	100.0 %	1.18	0.19
	Q3	72.46	146.95	0.55	138.92	110.21	100.0 %	1.26	0.19
	Q4	71.67	159.48	0.55	161.70	119.61	100.0 %	1.35	0.21
2010	Q1	74.76	175.16	0.55	171.05	131.37	97.6 %	1.30	0.20
	Q2	74.79	174.94	0.55	176.10	131.20	100.0 %	1.34	0.21
	Q3	72.46	162.25	0.55	176.29	121.69	96.0 %	1.45	0.24
	Q4	80.52	166.64	0.55	165.71	124.98	97.8 %	1.33	0.21

Table 5.7: Mean of Profit Index for selected OffPeak periods

Year	Quarter	Crude oil prices ($/Barrel)	Mean of customer price	Hedge ratio	Hedge price (S$/MWh)	Average cost (S$/MWh)	Selected data	Hedge price ratio	Mean of Profit Index
2004	Q1	29.13	81.20	0.56	94.24	60.90	75.9 %	1.55	0.32
	Q2	32.67	86.69	0.56	96.25	65.01	100.0 %	1.48	0.31
	Q3	36.45	88.26	0.56	95.73	66.19	95.6 %	1.45	0.30
	Q4	38.45	93.09	0.56	101.56	69.82	92.2 %	1.45	0.30
2005	Q1	40.29	92.33	0.51	101.29	69.24	85.4 %	1.46	0.28
	Q2	45.21	101.82	0.52	96.35	76.37	100.0 %	1.26	0.23
	Q3	55.19	110.94	0.52	117.38	83.20	99.7 %	1.41	0.26
	Q4	50.91	123.20	0.52	128.39	92.40	100.0 %	1.39	0.26
2006	Q1	54.52	126.43	0.51	140.70	94.82	97.5 %	1.48	0.27
	Q2	62.90	136.32	0.52	139.44	102.24	100.0 %	1.36	0.26
	Q3	63.51	140.74	0.51	147.90	105.56	95.3 %	1.40	0.26
	Q4	52.71	129.36	0.51	150.20	97.02	60.2 %	1.55	0.30
2007	Q1	51.86	119.05	0.51	134.66	89.29	74.4 %	1.51	0.23
	Q2	61.51	112.47	0.52	121.14	84.35	100.0 %	1.44	0.22
	Q3	69.60	123.24	0.44	137.25	92.43	97.1 %	1.48	0.20
	Q4	82.20	138.70	0.43	150.04	104.02	98.3 %	1.44	0.19
2008	Q1	90.21	152.66	0.43	161.80	114.49	97.9 %	1.41	0.18
	Q2	114.51	175.53	0.44	174.44	131.65	100.0 %	1.33	0.18
	Q3	113.33	185.50	0.43	183.25	139.13	100.0 %	1.32	0.18
	Q4	54.17	163.11	0.43	238.64	122.33	34.6 %	1.95	0.28
2009	Q1	40.16	114.89	0.44	167.14	86.17	12.2 %	1.94	0.28
	Q2	55.84	133.50	0.44	115.26	100.13	100.0 %	1.15	0.16
	Q3	72.46	136.90	0.43	138.92	102.68	99.5 %	1.35	0.18
	Q4	71.67	147.93	0.44	161.70	110.95	100.0 %	1.46	0.19
2010	Q1	74.76	160.10	0.44	171.05	120.07	100.0 %	1.42	0.19
	Q2	74.79	164.66	0.44	176.10	123.49	100.0 %	1.43	0.19
	Q3	72.46	152.58	0.44	176.29	114.43	99.8 %	1.54	0.20
	Q4	80.52	153.59	0.44	165.71	115.19	98.5 %	1.44	0.19

Table 5.8: Correlation coefficient of hedge price ratio and mean of Profit Index for selected periods

Year	Correlation coefficient of hedge price ratio and mean of Profit Index		
	Peak	Shoulder	OffPeak
2004	0.9998	0.9954	0.9897
2005	0.9993	0.9965	0.9969
2006	0.9998	0.9982	0.9692
2007	-0.5711	-0.2778	0.3380
2008	0.9985	0.9978	0.9971
2009	0.9878	0.9906	0.9918
2010	0.9964	0.9953	0.9943

Table 5.9: Estimated number of gencos and annual mean of Profit Index for selected periods

Year	Estimated number of gencos	Annual mean of Profit Index		
		Peak	Shoulder	OffPeak
2004	4	0.39	0.34	0.31
2005	4	0.39	0.32	0.26
2006	4	0.41	0.32	0.27
2007	5	0.29	0.24	0.21
2008	5	0.25	0.22	0.19
2009	5	0.20	0.20	0.18
2010	5	0.23	0.22	0.19

Table 5.10: Correlation coefficient of number of gencos and mean of Profit Index for selected periods

	Peak	Shoulder	OffPeak
Correlation coefficient of number of gencos and mean of Profit Index	-0.94	-0.98	-0.93

5.4 Conclusions

In this chapter, we use Cournot models to investigate the impact of bilateral contracts on the spot market. MCQ, SMQ, MCP, CP, total profit of the market and market power in the spot market are examined. The assumption that demand function is unchanged with the introduction of bilateral contracts is justified in our models.

There are three major results from our study. Firstly, we find several properties for the MCQ, SMQ, MCP and CP. When the bilateral contracts are introduced, MCQ may be increased and MCP may be decreased. We show that the MCQ is an increasing function of contract quantity. Also, the MCP and the SMQ are decreasing functions of contract quantity. We also show that MCQ with contracts is an upper bound of MCQ without contracts, and MCQ without contracts is an upper bound of SMQ. Moreover, we show that the MCP is reduced in the spot market with contracts. The variances of MCP with and without bilateral contracts are identical. However, the variance of CP is reduced with contracts. In the situation that the number of gencos goes to infinity, the MCQ and MCP are unchanged with and without contracts. Moreover, SMQ is a fixed portion of MCQ without contracts. In addition, we have that the allocation of fixed total contract quantity may not affect the MCQ, SMQ and MCP.

Secondly, we find several properties for total profit of the market. We derive the closed forms for total profit of the markets with and without contracts. We also show that the total profit of the market is reduced by the introduction of bilateral contracts if contract price is less than MC.

Thirdly, the impact of bilateral contracts on the market power is investigated. We use a conventional index, Lerner Index, to test the market power. Then, we propose a new index, called the Profit Index, to test the market power. The Lerner Index shows that market power is reduced by the introduction of bilateral contracts. By using the Profit Index, market power is an increasing function of contract price for a given contract quantity. Moreover, market power is a decreasing function of number of gencos. A numerical study is conducted using data of the Singapore electricity market from 2004 to 2010 to verify these two results.

Chapter 6

Conclusions and Future Research

The main purpose of this thesis is to investigate the effects of bilateral contracts. We are interested in the effects of bilateral contracts on controlling the price volatility and market power. In this chapter, we conclude the study by presenting and discussing the research results of Chapters 3, 4 and 5. Furthermore, possible directions for future research are presented.

6.1 Conclusions

In Chapter 3, we build mathematical models and analyze how the hedge price and quantity affect the uncertainties of MCP and CP. Variances are used to characterize the uncertainties of MCP and CP. We consider an unstable environment and assume that electricity supply is a discrete function. We assume that gencos make offers according to their production costs, generating unit availability and other related factors without strategic behavior. The production cost is uncertain due to uncertain fuel prices and possible breakdown of generating units. We also assume that demand is inelastic in our model. In a single period, gencos may consider a simple and fair strategy: having zero expectation on extra profit/cost caused by hedging price and quantity. In our model, we assume that gencos estimate the demand over and above the hedge quantity to be L and adjust their offer prices to balance the gain or loss with L for zero expectation. There are four major analytical results from our models. Firstly, we find that the variance of MCP increases when hedge quantity is assigned. Also, the variance of CP decreases when hedge quantity is assigned. Secondly, we find that the variances of MCP and CP do not have statistically significant relationships with the hedge price. Thirdly, we find that the variances of MCP and CP are decreasing functions of neutralizing quantity L. Fourthly, we find that the variance of MCP is an increasing function of hedge quantity. A numerical study is conducted using data from the Singapore electricity market from 2003 to 2010 to verify our model assumptions and the main results. The data are also used to conduct parameter estimation.

In Chapter 3, we develop the model and assume that the genco bids according to its marginal cost and does not consider bilateral contracts. To incorporate the competition behaviors of gencos, the SFE and Cournot models are adopted. In Chapter 4, we formulate the spot market by using SFE and Cournot models and examine the impact of bilateral contracts on the variances of MCP and CP. We assume that the production cost is quadratic and marginal cost is linear in both models. We also assume that demand function is linear with uncertainty. Under these assumptions, the variances of MCP and CP are decreasing functions of contract quantity in a competitive market. Even when the market is not competitive, bilateral contract can also reduce the variances of MCP and CP by setting contract quantity within a reasonable range. These two results hold in both SFE and Cournot models. Real data from the Singapore electricity market from 2003 to 2010 are used to verify our findings. The results of the numerical study support our models.

The price volatility is studied in Chapters 3 and 4. In Chapter 5, we study Cournot models to investigate the impact of bilateral contracts on the spot market. The MCQ, SMQ, MCP, CP, profit of the market and market power in the spot market are examined closely. We assume that the production cost is linear in Cournot models. We also assume the demand function to be linear with uncertainty. There are three features in this chapter.

Firstly, we assume that demand function is changed with the introduction of bilateral contracts in our models. The analytical results show that our models are identical to those models with unchanged demand functions. This finding provides good justification for the assumption that the demand function is unchanged with the introduction of bilateral contracts.

Secondly, we find some properties for the MCQ, SMQ, MCP, CP and profit of the market. When the bilateral contracts are introduced, MCQ may be increased and MCP may be decreased. We show that the MCQ is an increasing function of contract quantity. Also, the MCP and the SMQ are decreasing functions of contract quantity. We also show that MCQ with contracts is an upper bound of MCQ without contracts, while MCQ without contracts is an upper bound of SMQ. Moreover, we show that the MCP is reduced in the spot market with contracts. We also show that the variances of MCP are identical with and without bilateral contracts. However, the variance of CP is reduced with contracts. In addition, we find that the allocation of total contract quantity may not affect the MCQ, SMQ and MCP; that is, the allocation of fixed total contract quantity has no relationship with the MCQ, SMQ and MCP. Besides, we find several properties for the profit of the market. We derive the closed forms for total profit of the market with and without contracts. We also show that the total profit of the market is reduced by the introduction of bilateral contracts if contract price is less than MC.

Thirdly, the impact of bilateral contracts on the market power is investigated. We first use a conventional index, Lerner Index, to test the market power. This Lerner Index shows that market power is reduced by the introduction of bilateral contracts. We then propose another index which is defined as the ratio of profits with and without competition. We call this index the Profit Index. By using this Profit Index, we find that market power is an increasing function of contract price subject to a given contract quantity. Several numerical studies are conducted using data of the Singapore electricity market to verify our analytical results.

6.2 Possible future research

There are several possible extensions of this thesis. One extension is to consider different risk measurement tools, such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). Another extension is to relax existing assumptions. Considering multi-period models is also a possible extension. With multi-period models, we can use game theory to study the interaction of market participants.

6.2.1 Different measurements on price volatility

In this thesis, we use variance to measure the uncertainty. We would like to consider other uncertainty measures on the market price. From the viewpoint of the government, we care not only about the uncertainty of market price but also the risks faced directly by market participants. VaR is a methodology developed by the financial industry. It measures the expected maximum loss over a certain time horizon within a given confidence interval. CVaR is defined as the conditional expectation of losses given that the loss exceeds a threshold value (Alexander et al., 2006). Thus, VaR and CVaR may be better for measuring the risks of market participants. VaR and CVaR can be used to measure the uncertainties and risks of MCP and CP in future studies.

6.2.2 Relaxation of assumptions

In Chapter 3, we assume that the revenue or cost arising from hedging will be neutralized by selling the next L quantity. However, in the real world, this revenue or cost may or may not be neutralized. Alternatively, it will be neutralized in other ways. Thus, we are interested in different neutralizing methods as well. For example, gencos may neutralize their loss without neutralizing their gain from contracts.

In Chapter 4, we assume that the gencos are symmetric and the supply functions they submit are linear. These two assumptions may not reflect the situations in the real world. For example, the gencos submit offers (price-quantity pairs) instead of linear supply functions in the real world. Hence, we would like to relax those assumptions in the future work.

In Chapter 5, we assume production cost to be linear. In the future, we may consider quadratic production cost functions which are widely used in related studies (Niu et al., 2005; Bushnell, 2007). Instead of using the Cournot model, we may also consider SFE models (Klemperer and Meryer, 1989). These models reflect more characteristics of the real electricity market than Cournot models. For example, the gencos submit offers (price-quantity pairs) in the real market. We can formulate these offers as step functions. In SFE models, all gencos maximize their profits by choosing optimal supply functions, unlike Cournot models in which gencos only compete on quantity. As a result, SFE models can capture the characteristics of the offers better than Cournot models.

6.2.3 Multi-period problem

In this thesis, we only consider single-period in our models. However, the decisions made by gencos are usually for multi-period situations. Moreover, gencos may not be able to neutralize their profits within one period. They may neutralize their profits in multiple periods. In addition, gencos competing in a market may deliberately lose money during certain periods in order to monopolize the market. Then, they can neutralize the lose in the future. Thus, multi-period models provide an interesting future research direction.

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